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Regulation and the Emergence of Temporary  
Work Agencies**

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# THRESHOLD EFFECTS OF DISMISSAL PROTECTION REGULATION AND THE EMERGENCE OF TEMPORARY WORK AGENCIES

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## Abstract

Labour market regulations aimed at enhancing job-security are dominant in several OECD countries. These regulations seek to reduce dismissals of workers and fluctuations in employment. The main theoretical contribution is to gauge the effects of such regulations on labour demand across establishment sizes. In order to achieve this, we investigate an optimising model of labour demand under uncertainty through the application of real option theory. We also consider other forms of employment which increase the flexibility of the labour market. In particular, we are modelling the contribution of temporary employment agencies (*Zeitarbeit*) allowing for quick personnel adjustments in client firms. The calibration results indicate that labour market rigidities may be crucial for understanding sluggishness in firms' labour demand and the emergence and growth of temporary work.

**Keywords:** Labour Demand, Dismissal Protection Legislation, Firing Costs, Real Options, Temporary Work Agencies, Temporary Employment

**JEL-Classification:** J23, J58, D81

## 1. Introduction

In many continental European countries unemployment appears to reside at a persistently high level, with no improvement in sight. Therefore, protection of workers from dismissals has become an important topic of labour markets reforms in many European countries. According to the World Bank *Doing Business* database, countries vary greatly with respect to the flexibility of labour market regulations.<sup>1</sup> These regulations can be provided through legislation, collective bargaining agreements or judicial practices and court interpretations of legislative provisions. According to the World Bank, for example, severance pay in Germany is set at 66.7 weekly wages, in the Netherlands at 16.0 weekly wages, in the UK at 33.5 weekly wages, and Portugal requires 98.0 weekly salaries as the standard compensation. On the contrary, the corresponding number for the U.S. is 0.0. Given these differences the pros and cons of deregulating labour markets are at the heart of the employment debate in many countries.

An important characteristic of dismissal protection laws or collective agreements in advanced economies is that rules for dismissing redundant workers are differentiated by establishment size and the provisions are more stringent above certain employee thresholds. In Germany, the threshold in the “Protection Against Dismissal Act” (*Kündigungsschutzgesetz*) was changed several times. During the 1990s, the threshold was changed twice, once from 5 to 10 (full-time equivalent) employees in October 1996 by the then chancellor Helmut Kohl and then back again to 5 employees in January 1999 under chancellor Schröder. Finally, in January 2004 the threshold was moved once again from 5 to 10 employees. The size exemption criteria apply to establishments, not firms. An establishment is a production unit at a single location which can financially and/or legally belong to a larger firm. Establishments below the threshold are allowed to operate under the far less stringent rules of the German Civil Code (*Bürgerliches Gesetzbuch*). The corresponding Austrian threshold level for application of dismissal protection is 5 workers, while no such thresholds currently exist in the UK and the Netherlands.<sup>2</sup>

The potential importance of the 5-employee-threshold vs. 10-employee-threshold in the “Protection Against Dismissal Act” results from the fact that Germany’s economy is dominated by small and

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<sup>1</sup> The World Bank *Doing Business* scoreboard on the flexibility of labour regulations and their enforcement is available at [www.doingbusiness.org/ExploreTopics/HiringFiringWorkers/](http://www.doingbusiness.org/ExploreTopics/HiringFiringWorkers/). The table provides five indicators for a worker in a large manufacturing firm who has been with the company for many years. (1) Difficulty of hiring a new worker (Difficulty of Hiring Index); (2) restrictions on expanding or contracting the number of working hours (Rigidity of Hours Index); (3) difficulty and expense of dismissing a redundant worker (Difficulty of Firing); (4) an average of the three indices (Rigidity of Employment Index), and (5) cost of a redundant worker, expressed in weeks of wages (Firing Costs). Higher values in the table indicate more rigid regulations. Also see Botero et al. (2004). The OECD has also published indices of employment protection, again showing less protection in English-speaking countries [see OECD (2004)].

<sup>2</sup> Apart from national legislation, the EU *acquis* also sets thresholds proportional to firms’ employment level, by which individual dismissals become collective dismissals with corresponding effects on firing costs (up and down, depending on the regulation on each type of dismissal in each country).

medium-sized firms, the *Mittelstand*, which has often been described as the backbone of the German economy.<sup>3</sup>

Our aim is to model the effects of such thresholds upon labour demand. To this end, we construct a real options model of labour demand with threshold effects. In particular, we are modelling the contribution of temporary employment agencies which represent a key response to the flexibility needs of firms and help firms to cope with fluctuations in demand in a more global and fast-changing environment. Temporary agency work can broadly be defined as a temporary employment relationship between a temporary work agency, which is the employer, and a worker, where the latter is assigned to work for, and under the control of a firm making use of his/her services.

The remainder of the paper is structured as follows. Section 2 develops the baseline theoretical model of employment dynamics and demonstrates the implications of various policy reforms with illustrative numerical examples. Section 3 extends the model to allow for the contribution of temporary employment agencies which represent a key response to the flexibility needs of firms and help them to cope with fluctuations in demand in a fast-changing environment. In Section 4, a further perspective on the adjustment of employment is obtained by simulating the dynamic employment patterns of the models. Section 5 concludes. Two appendices at the end of the paper collect some proofs and technical derivations which are rather involved. Readers who are not interested in the nuts and bolts of the derivations can skip the appendices without losing the main argument of the paper.

## 2. The Baseline Modelling Framework

In this section we construct a real options model for employment under uncertainty. The stochastic framework contains the threshold effect induced by the “Protection Against Dismissal Act” to advance our understanding of the impact of the institutional setting. Like other real option models of this type, the optimal employment policy is a trigger strategy such that hirings and firings are initiated when the marginal product of labour reaches a critical threshold.<sup>4</sup> We believe this to be an appropriate framework for understanding the impact of threshold levels for application of dismissal protection on employment, while still yielding tractable results. We first characterise the optimal employment strategy of an imperfect competitive firm subject to idiosyncratic shocks and firing and hiring costs, holding wages and productivity constant. The starting point is the Cobb-Douglas production function

$$(1) \quad Y_t = AK^\alpha L_t^{1-\alpha}, \quad 0 < \alpha < 1,$$

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<sup>3</sup> In 2001 about two-third of all German establishments had 1 to 5 employees. See EUROSTAT (2001) for further details. According to the German Statistical Office even 91 percent of all German firms had 1-9 employees in 2006 (see <http://www.destatis.de/basis/d/insol/unternehtab2.php>).

<sup>4</sup> In its methodological approach, the model comes within the scope of the real options literature which has developed rapidly over the past decade. Reviews of this burgeoning literature are provided in Copeland and Antikarov (2001), Copeland and Tufano (2004), Coy (1999) and Dixit and Pindyck (1994).

where  $K$  is the constant capital stock,  $L_t$  is the employment level,  $\alpha$  is a parameter determining the shares between capital and labour in production, and  $A$  represents the level of technology. It is assumed that the firm faces an isoelastic demand function subject to multiplicative demand shocks

$$(2) \quad p = Y_t^{(1-\psi)/\psi} Z_t, \quad \psi \geq 1,$$

where  $p$  denotes the price,  $Y_t$  is real output,  $Z_t$  denotes the multiplicative stochastic demand shock, and  $\psi$  is an elasticity parameter that takes its minimum value of 1 under perfect competition. Therefore, the profits at  $t$ ,  $\Pi_t$ , measured in units of output, are defined as

$$(3) \quad \Pi_t = A^{1/\psi} Z_t K^{\alpha_1} L_t^{\alpha_2} - wL_t - C(M_t, L_t),$$

where  $\alpha_1 = \alpha/\psi$  and  $\alpha_2 = (1 - \alpha)/\psi$ ,  $wL_t$  denotes the total wage bills paid by the firm,  $M_t$  represents gross employment changes due to hiring or firing from employees, and  $C(\cdot)$  are the total employment adjustment expenditures.<sup>5</sup> For tractability purposes we choose the following smooth cost function for the threshold effects containing asymmetric fixed, proportional, and convex costs of adjusting employment in either direction [see Nilsen et al. (2003)]:

$$(4) \quad C(M_t) = \begin{cases} c_h + p_h M_t + \frac{1}{2} \gamma_h M_t^2 & \text{for } M_t > 0, \\ 0 & \text{for } M_t = 0, \\ c_{f_1} + \frac{c_{f_2}}{1 + e^{-c_0(L_t - l)}} - p_f M_t + \frac{1}{2} \gamma_f M_t^2 & \text{for } M_t < 0. \end{cases}$$

There is an economic meaning behind these three cost components. (1) Hiring and firing employees incur some proportional positive unit costs of hiring and firing,  $p_h$  and  $p_f$ , respectively.<sup>6</sup> Firing employees does generate some positive costs per employee:  $-p_f M_t > 0$  for  $M_t < 0$ . The positive unit costs of hiring and firing also reflect the (partial) irreversibility of employment changes; (2) the convex cost functions reflect the adjustment and disruptions to production processes; in case of asymmetric convex costs marginal cost of hiring are not the same as the marginal costs of firing; (3) the

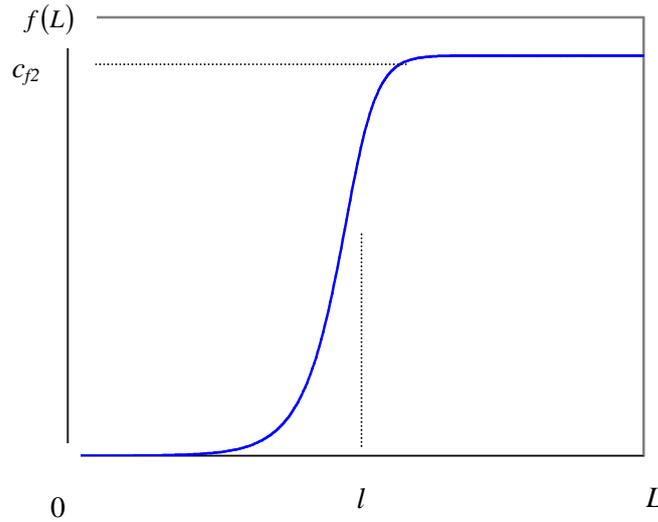
<sup>5</sup> In this paper, for the sake of simplicity, we specify adjustment costs as a function of gross employment changes, consistent with many papers in the literature [see, for example, Abowd and Kramarz (2003)]. For specifications based on both gross and net employment flows, see Hamermesh (1995).

<sup>6</sup> In the model we abstract from the choice of hours worked. For a real options framework with overtime and short time work, see Chen and Funke (2004).

fixed costs of hiring and firing are related to advertising and screening and are set up to a point independent of the number of people hired. The costs also include fixed costs of legal consultation and disputes in case of firings. In addition to explicit costs, a change in the level of employment is likely to involve implicit costs in terms of temporary productivity losses; (4) moreover, there are no costs as long as no hirings/firings are made, or equivalently,  $C(0) = 0$ .

The novelty of our model is that we allow for heterogeneity among firms by formalizing the threshold levels for application of dismissal protection which exist in many countries. We assume that the fixed cost of firing can be depicted by a three-parameter logistic function,  $c_{f_2} / (1 + e^{-c_0(L_t - l)})$ , where  $l$  denotes the threshold level for application of dismissal protection,  $c_0$  is a scale parameter indicating the speed of such transition to the values of  $c_{f_1}$  and  $c_{f_2}$ , which are the final size of fixed costs of firing when  $L$  is greater than  $l$ . By using this logistic function, changes in legislation can be accounted for in great detail. To see this, Figure 1 depicts the shape of the adjustment costs as a function of  $l$  and  $c_f$ .

**Figure 1: The fixed cost function**  $f(L) = c_{f_2} / (1 + e^{-c_0(L_t - l)})$



A simple fixed cost of hiring,  $c_h$ , is assumed for the hiring decision. Note that all parameters in equation (4) are assumed to be positive. Employment evolves according to

$$(5) \quad \frac{dL_t}{dt} = M_t - \delta L_t,$$

where  $\delta$  represents the deterministic quit rate. We assume that the multiplicative demand shock follows the geometric Brownian motion,

$$(6) \quad dZ_t = \eta Z_t dt + \sigma Z_t dW_t,$$

where  $W_t$  is a standard Wiener process with independent, normally distributed increments,  $\eta$  is the deterministic drift parameter, and  $\sigma$  is the variance parameter.<sup>7</sup>

The next task is to characterise the objective function of the firm. The firm chooses its optimal level of gross employment changes,  $M_t$ , over time to maximise the intertemporal value of profits, subject to the employment stock accumulation [equation (5)] and the geometrical Brownian motion [equation (6)]. More precisely, we assume that the firm maximises the present discounted value of its stream of current and expected future profits, defined as:

$$(7) \quad V = \max_M E \left[ \int_{t=0}^{\infty} [A^{1/\psi} Z_t K^{\alpha_1} L_t^{\alpha_2} - wL_t - C(M_t)] e^{-rt} dt \mid Z = Z_0, K = K_0 \right], \text{ s.t. (5) and (6),}$$

where  $E[\cdot|\Omega_t]$  denotes the mathematical expectation given the information set available to the firm at period  $t$ ,  $\Omega_t$ ,  $r > 0$  is the interest rate and  $wL_t$  is the wage bill. Applying Ito's Lemma, the stochastic nature of this optimization problem requires the solution to the following Bellman equation:

$$(8) \quad rV = \max_M E \left[ A^{1/\psi} Z K^{\alpha_1} L^{\alpha_2} - wL - C(M) + V_L (M - \delta L) + \eta Z V_Z + \frac{1}{2} \sigma^2 Z^2 V_{ZZ} \right],$$

where  $V$  represents the intertemporal value of the firm.<sup>8</sup> Intuitively, equation (8) can be interpreted as follows: Should the option to hire be tradable and its risk diversifiable, then the expected value has to be equal to the foregone revenue from interest ( $rV$ ). The first-order conditions for gross employment changes yield

$$(9) \quad \pm p_{h/f} + \gamma_{h/f} M = v,$$

where  $v = V_L$ . Note that the fixed costs of employment adjustment disappear in equation (9). However, the fixed costs of adjustment will enlarge the inaction area due to the fact that the firm only undertakes employment changes if a non-negative profit arises after deducting the fixed costs. It can be shown (see Appendix A) that hirings and firings occur when

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<sup>7</sup> At this juncture an additional remark about this stochastic process is in place. A Brownian motion with a drift is the limit of a random walk with uneven probabilities for negative and positive changes. A positive (negative) drift implies that positive (negative) changes are more likely to occur than negative (positive) changes. The drift rate thus represents the bias of uncertainty.

$$(10) \quad M \geq \sqrt{\frac{2c_h}{\gamma_h}} \geq 0$$

and

$$(11) \quad M \leq -\sqrt{2\left(c_{f_1} + \frac{c_{f_2}}{(1+e^{-c_0(L-l)})}\right)} / \gamma_f \leq 0.$$

The boundaries of the inaction area satisfy:

$$(12) \quad v \geq p_h + \sqrt{2c_h\gamma_h} \text{ for hiring thresholds,}$$

and

$$(13) \quad v \leq -p_f - \sqrt{2\left(c_{f_1} + \frac{c_{f_2}}{(1+e^{-c_0(L-l)})}\right)} \gamma_f \text{ for firing thresholds.}$$

The upper threshold can be derived by finding the value of  $v$  at which an additional worker generates non-negative profits. The lower threshold is found in a similar fashion. It is obvious that the higher the fixed costs of hiring/firing  $(c_h, c_f)$ , the greater is the number of hiring/firing in these employment decisions and the wider is the inaction area. The firm does not hire/fire employees for the boundaries of  $v = \pm p_{h/f}$ ; it waits until the numbers of hiring/firing reaching certain values to cover the non-trivial fixed costs of hiring/firing so that equations (9) are satisfied. The adjustment speed-related parameters  $(\gamma_h, \gamma_f)$  also affect the numbers of hiring/firing. With a very small adjustment cost, the adjustment speeds increase and the firm tends to hire/fire more employees. The higher adjustment speeds due to smaller values of  $(\gamma_h, \gamma_f)$  also imply that the values of  $v$  do not deviate substantially from outside of  $v = \pm p_{h/f}$  – a smaller inaction area.<sup>9</sup>

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<sup>8</sup> In the case of reversible hiring decisions, the effect of future profits does not occur because earlier hirings can be withdrawn at any time. Thus, it is sufficient to consider the marginal product of labour at present time  $t$  only.

<sup>9</sup> Note that contrary to the “now or never” decision in a traditional modelling framework with instantaneously and costlessly adjustable factors of production, the firm must choose the optimal time to fire or hire. This means that at every moment it must compare the continuation value, i.e. the value of the option when kept unexercised, and the value of an immediate firing or hiring decision.

For the levels of  $M$  falling into the regime of  $-\sqrt{2\left(c_{f_1} + \frac{c_{f_2}}{(1+e^{-c_0(L-l)})}\right)} / \gamma_f < M < \sqrt{2c_h/\gamma_h}$ , the firm does not hire or fire employees simply because the benefits from employment changes are not large enough to cover the fixed costs of hiring or firing, or even the proportional unit costs of hiring or firing. We can consider that  $p_h + \sqrt{2c_h\gamma_h}$  as effective marginal hiring costs when considering mass-hiring, and  $p_f + \sqrt{2\left(c_{f_1} + \frac{c_{f_2}}{(1+e^{-c_0(L-l)})}\right)}\gamma_f$  as effective firing costs.

The procedure in the Appendix A removes the nonlinear terms related to adjustment costs in Bellman equations and transfers them into parts of the effective hiring and firing costs. Thus, we have the following analytically solvable differential equation for the boundaries of (mass-) hiring/firing decisions:

$$(14) \quad (r + \delta)v = \alpha_2 A^{1/\psi} ZK^{\alpha_1} L^{\alpha_2-1} - w - \delta v_L L + \eta Z v_Z + \frac{1}{2} \sigma^2 Z^2 v_{ZZ},$$

where  $v_Z = V_{LZ}$ ,  $v_L = V_{LL}$  and  $v_{ZZ} = V_{LZZ}$ . Equation (14), subject to the boundary conditions of equations (12) and (13), can be solved to obtain the hiring and firing thresholds ( $Z_H$  and  $Z_F$ ) for the corresponding values of demand shocks.

After some algebra it can be shown (see Appendix B) that the particular solutions for  $v$  denoting the intertemporal marginal value of employees when no hiring and firing occurs takes the form

$$(15) \quad v^P = \frac{\alpha_2 A^{1/\psi} ZK^{\alpha_1} L^{\alpha_2-1}}{r + \alpha_2 \delta - \eta} - \frac{w}{r + \delta},$$

and the general solutions for  $v$  representing the value of the real options to hire and fire are denoted by

$$(16) \quad q^G = -B_1 \left( ZK^{\alpha_1} L^{\alpha_2-1} \right)^{\beta_1} + B_2 \left( ZK^{\alpha_1} L^{\alpha_2-1} \right)^{\beta_2},$$

where  $B_1$  and  $B_2$  are two unknown positive variables to be determined by the boundary conditions – the value-matching and smooth-pasting conditions – and  $\beta_1$  and  $\beta_2$  are the positive and negative characteristic roots of the following equation, respectively:

$$(17) \quad r + \delta + \delta\beta(\alpha_2 - 1) - \eta\beta - \frac{1}{2} \sigma^2 \beta(\beta - 1) = 0.$$

The term  $B_1(ZK^{\alpha_1}L^{\alpha_2-1})^{\beta_1}$  is usually interpreted as the real option to hire and the term  $B_2(ZK^{\alpha_1}L^{\alpha_2-1})^{\beta_2}$  is considered as the real option to fire. The value-matching and smooth-pasting conditions follow, and determine the thresholds of hiring and firing.<sup>10</sup> Both conditions ensure that along the boundaries the firm is indifferent at the margin between an adjustment at date  $t$  and waiting  $dt$  to make the adjustment at date  $t + dt$ . The value-matching conditions are:

$$(18) \quad \frac{\alpha_2 A^{1/\psi} Z_H K^{\alpha_1} L^{\alpha_2-1}}{r + \alpha_2 \delta - \eta} - \frac{w}{r + \delta} + B_2 (Z_H K^{\alpha_1} L^{\alpha_2-1})^{\beta_2} = p_h + \sqrt{2c_h \gamma_h} + B_1 (Z_H K^{\alpha_1} L^{\alpha_2-1})^{\beta_1}$$

and

$$(19) \quad - \left[ \frac{\alpha_2 A^{1/\psi} Z_F K^{\alpha_1} L^{\alpha_2-1}}{r + \alpha_2 \delta - \eta} - \frac{w}{r + \delta} \right] + B_1 (Z_F K^{\alpha_1} L^{\alpha_2-1})^{\beta_1} \\ = p_f + \sqrt{2 \left( c_{f_1} + \left( \frac{c_{f_2}}{1 + e^{-\alpha_0(L-l)}} \right) \right) \gamma_f} + B_2 (Z_F K^{\alpha_1} L^{\alpha_2-1})^{\beta_2}.$$

The smooth-pasting conditions take the forms:

$$(20) \quad \frac{\alpha_2 A^{1/\psi} K^{\alpha_1} L^{\alpha_2-1}}{r + \alpha_2 \delta - \eta} + \beta_2 B_2 Z_H^{\beta_2-1} (K^{\alpha_1} L^{\alpha_2-1})^{\beta_2} = \beta_1 B_1 Z_H^{\beta_1-1} (K^{\alpha_1} L^{\alpha_2-1})^{\beta_1}$$

and

$$(21) \quad - \frac{\alpha_2 A^{1/\psi} K^{\alpha_1} L^{\alpha_2-1}}{r + \alpha_2 \delta - \eta} + \beta_1 B_1 Z_F^{\beta_1-1} (K^{\alpha_1} L^{\alpha_2-1})^{\beta_1} = \beta_2 B_2 Z_F^{\beta_2-1} (K^{\alpha_1} L^{\alpha_2-1})^{\beta_2}.$$

Equations (18) - (21) consist of a non-linear system of four equations with four unknown variables,  $Z_H$ ,  $Z_F$ ,  $B_1$  and  $B_2$ . Generally, numerical methods have to be adopted because closed-form solutions cannot be derived. In order to develop a “feel” for the model and to “draw a map” of the labour demand sensitivity to various structural characteristics of the environment in which firms operate, we calibrate parameters as follows. We interpret periods as years and annual rates are used where applicable. Where

possible, parameter values are drawn from empirical studies.<sup>11</sup> Our base parameters are  $\sigma = 0.2$ ,  $\eta = 0$ ,  $r = 0.05$ ,  $\delta = 0.05$ ,  $p_h = 0.02$ ,  $c_h = 0.06$ ,  $\gamma_h = 0.01$ ,  $p_f = 0.05$ ,  $c_{f_1} = 0.01$ ,  $c_{f_2} = 0.29$ ,  $\gamma_f = 0.5$ ,  $c_0 = 100.0$ ,  $l = 5.0$ ,  $K = 6.0$ ,  $\alpha = 0.3$ ,  $\psi = 1.5$  and  $A = 1$ . In practice, measuring product market competition is a complex task. In our baseline parameter specification the price elasticity of demand parameter is set at  $\Psi = 1.50$  as in Bovenberg et al. (1998). The deterministic drift term  $\eta$  has been set to zero to avoid any “bias in uncertainty”. The labour share  $1 - \alpha$  (profit share  $\alpha$ ) is 0.7 (0.3). For simplicity, we normalise capital such that  $K = 6.0$ . This does not affect the qualitative results. We set  $A = 1$  without loss of generality. The baseline threshold level for application of dismissal protection is  $l = 5$ . The choice of the remaining labour adjustment cost parameters can be explained as follows. Beyond the threshold  $l = 5$ , the effective firing costs should reach 0.6.<sup>12</sup> Our benchmark value of

$$p_f + \sqrt{2 \left( c_{f_1} + \frac{c_{f_2}}{1 + e^{-c_0(L-l)}} \right) \gamma_f} \text{ beyond } l = 5 \text{ is } p_f + \sqrt{2c_f\gamma_f} = 0.05 + \sqrt{2 \times 0.3 \times 0.5} = 0.5977 \approx$$

0.6; the effective hiring costs  $p_h + \sqrt{2c_h\gamma_h} = 0.02 + \sqrt{2 \times 0.06 \times 0.01} = 0.0546$  are also in the range of 0.06 as suggested by Bentolila and Bertola (1990). The corresponding hiring and firing  $M$ 's are

$$M = \sqrt{\frac{2c_h}{\gamma_h}} = 3.46410 \text{ for hiring and } M = -\sqrt{2 \left( c_{f_1} + \frac{c_{f_2}}{1 + e^{-c_0(L-l)}} \right) \gamma_f} = -1.09545 \text{ for firing after } L > l$$

$= 5$ . Figure 2 and 3 provide a graphical description of the pattern of employment adjustment for  $l < 5$  vs.  $l < 10$ .

The intuitive graphs dichotomize the space spanned by  $Z$  shocks into action and inaction areas. In the inaction area the marginal reward for changing employment is insufficient: neither hiring nor firing is optimal. The comparison of Figure 2 and Figure 3 reveals what is happening when countries try to deregulate labour markets by shifting the threshold where dismissal protection will be effective from  $l = 5$  to  $l = 10$ . The widening of the inaction area beyond the threshold indicates that the “Protection Against Dismissal Act” reduces the propensity to hire and fire with respect to the unregulated world.<sup>13</sup> The direct cost of employment protection makes adjustment of labour more expensive, which tends to lower firms’ willingness to hire. On the other hand, effective legal protection of existing employment

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<sup>10</sup> The value-matching conditions involve the value function, while the smooth-pasting conditions concern its first-order derivatives.

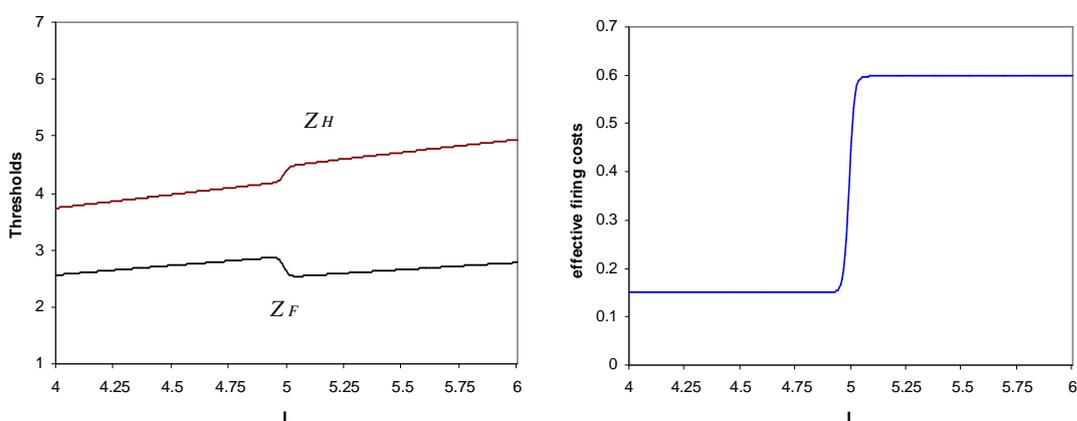
<sup>11</sup> It should be acknowledged that despite efforts to rely on multiple sources and datasets, there is inevitably an arbitrary and subjective aspect to some dimensions of the calibration. In particular, it is difficult to ascertain and quantify the extent of enforcement of statutory restrictions across firm sizes. We suggest taking an eclectic approach to capturing key economic features of policy interest. The basic idea is to choose coefficients that seem reasonable based on economic principles, available econometric evidence, and an understanding of the functioning of the economy, and then to look at how sensible the responses of the real options model are.

<sup>12</sup> Our parameters convey the message in Bentolila and Bertola (1990). Their estimated firing costs for Germany are in the range of 0.562 to 0.750, and their hiring cost estimate (excluding on-the-job-training) for Germany is 0.066 of the average annual wage.

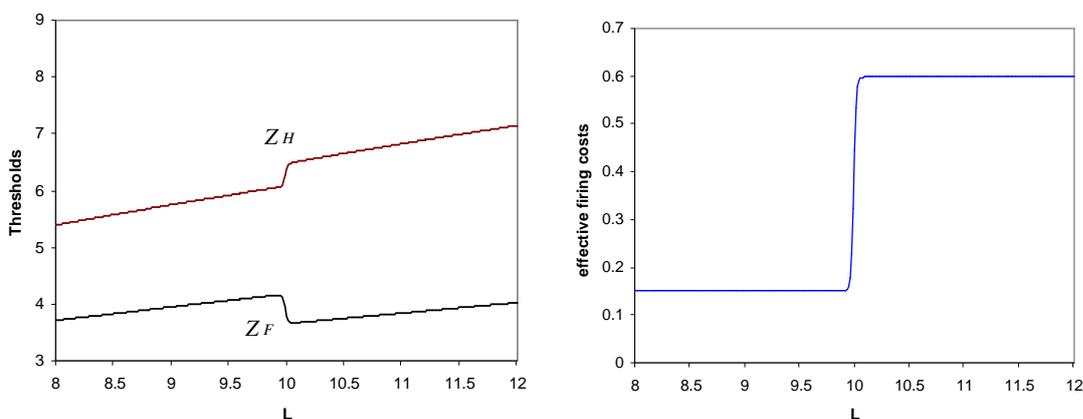
<sup>13</sup> The widening of the gap is consistent with the empirical evidence presented in Bauer et al. (2004) analysing worker flows in German establishments from March 1995 to March 1998 who have undergone periods of protection and non-protection as a consequence of the repeated changes in the German dismissal legislation.

relationships lowers the occurrence of firing during recessions. As firing and hiring incentives work in opposite directions, the impact of tighter or softened adjustment costs for labour is theoretically ambiguous. Bentolila and Bertola (1990) and Bertola (1992) have demonstrated in real option models that the overall impact depends, inter alia, upon the size of the adjustment costs, the functional forms and the discount rate. Our numerical findings verify the conjecture that the overall effect is a reduction of the speed of adjustment to shocks, but for fixed wages  $w = \bar{w}$  the net effect turns out to be mostly positive. In other words, the simulations imply that firing costs have more of an effect on the firing decision than on the hiring decision, thereby increasing long-term employment.<sup>14</sup>

**Figure 2: The Effects of Dismissal Protection Regulation with Exempted Establishment  $L < 5$**



**Figure 3: The Effects of Dismissal Protection Regulation with Exempted Establishment  $L < 10$**



Focusing on the results close to the thresholds, the calibration results indicate that the anticipation of future firing costs may have current effects for a hiring firm even when the more stringent firing regime beyond the threshold is absent at the time of decision making. Elaborating on this idea and using our formal theoretical model of labour demand decisions under uncertainty, the results in Figure 2 and 3 indicate that *latent* legal constraints can affect firms' employment policy even when these firing

<sup>14</sup> Pissarides (2001) has outlined another mechanism. He has shown that dismissal protection might increase welfare by providing insurance against unemployment.

constraints are currently slack. The numerical calibrations elegantly demonstrate this, as the outcome of a forward-looking behaviour by the small firm that expects future legal constraints to bind, resulting in current employment decisions to be a function both of the current legal framework but also expectations about their more stringent future path after growing beyond the threshold.

The main conclusion of the previous analysis is that firing costs beyond some thresholds tend to reduce both dismissals and hirings. Therefore, the net impact upon employment is theoretically indeterminate.

This notwithstanding, the effects of employment protection are likely to be different across firm size.

We are aware of the many caveats that such ceteris-paribus comparison can arise. A first concern is that tighter employment regulations may diminish company's ability to cope with a rapidly changing environment driven by globalisation, technical progress, and organisational change. Caballero et al. (2004) have demonstrated that job security legislation hampers the creative-destruction process and lowers productivity growth. The clear and robust result is that tight job security regulations lower annual productivity growth somewhere between 0.8 and 1.2 percent. Samaniego (2006) has demonstrated in a theoretical model that high firing costs slow the diffusion of new technologies via the mobility of entrepreneurial resources. Therefore firing costs are particularly detrimental in industries in which the rate of technical progress is rapid. Autor et al. (2007) have tested the theoretical prediction that firms facing firing costs will curtail hiring below efficient levels in a boom and retain unproductive workers in a recession, both of which should affect productivity, using plant-level data for the US. The microdata indicate that an increase in adjustment costs decreases efficiency (TFP). Finally, Acemoglu et al. (2006) and Colecchia (2002) have also demonstrated in an independent literature that a more competitive institutional setting will contribute to a more innovative and dynamic economy through thriving entrepreneurial activity. Although these studies did not use the same methodology as in this paper, their results which are based on theoretical models and regressions reveal a similar story as the one below.

In order to gauge the costs of job security provisions with endogenous creative destruction processes, we adapt our baseline real options model and assume that the level of aggregate productivity  $A$  is a function of total effective employment adjustment costs ( $TEEA$ ), i.e. the magnitude of the inaction area:

$$(22) \quad A = \frac{1}{1 + A_0 \times TEEA},$$

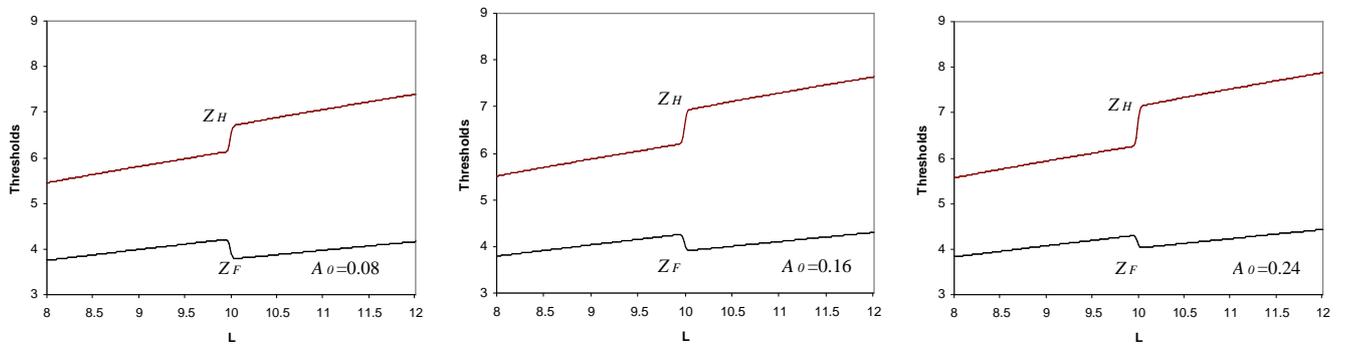
$$\text{where } TEEA = p_h + \sqrt{2c_h\gamma_h} + p_f + \sqrt{2\left(c_{f1} + \frac{c_{f2}}{1 + e^{-c_0(L-l)}}\right)\gamma_f}.$$

**Table 1: The Value of Productivity,  $A$ , for Different  $A_0$  and Regulation Regimes ( $l=10$ )**

	$L < 10$	$L \geq 10$
$A_0=0.08$	0.9917	0.9504
$A_0=0.16$	0.9835	0.9055
$A_0=0.24$	0.9755	0.8646

The implications of employment-protection-induced productivity changes upon the inaction area for  $l = 10$  are summarised graphically in Figure 4 for  $A_0 = 0.08$ ,  $A_0 = 0.16$ , and  $A_0 = 0.24$ , respectively.<sup>15</sup> Taking this productivity impact into account implies that contrary to the baseline model labour market regulations could indeed be a barrier to employment.

**Figure 4: The Inaction Area with Employment-Protection-Induced Productivity Changes for  $l=10$**



A second concern is that wages in the baseline model have been fixed although much of the debate of persistent unemployment in European economies has focused on the wage formation systems. Caballero and Hammour (1998) have shown that dismissal protection legislation like any other mandatory employment protection measure creates a hold-up problem enabling insiders (incumbent workers) to bid up wages once they are employed.<sup>16</sup> In other words, firing costs make it difficult for firms to fire workers, so firms hesitate to hire them in the first place, strengthening the hand of unions bargaining with firms to set a wage. Contrary to the traditional literature holding wages fixed and looking at the employment effect of different degrees of job security provisions, we therefore adapt our model to an insider-outsider mechanism where firing costs increase the bargaining power of incumbent workers [see, e.g., Díaz-Vázquez and Snower (2003) and Lindbeck and Snower (1988)].<sup>17</sup> Explicitly modelling

<sup>15</sup> In the simulations we assume that the extra gains in productivity will not be eroded by higher wages. The deregulated labour market and the threat that jobs in advanced economies could move abroad may help to hold wages down.

<sup>16</sup> In countries with higher firing costs a large share of workers with fixed-term contracts tends to insulate insiders (permanent workers) from adjustment, thereby increasing their bargaining power.

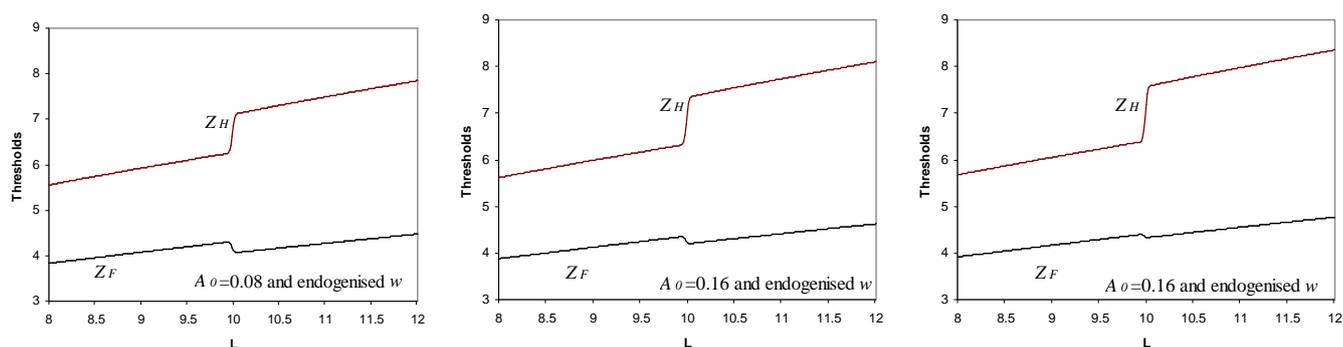
<sup>17</sup> Lazear (1990) has claimed that the non-wage labour costs arising from mandatory dismissal protection will be offset by an efficient contract or bargaining process (in the sense that they do not influence equilibrium unemployment, just equilibrium wages). However, labour markets are imperfect and wages are fairly rigid. It is therefore unlikely that dismissal costs are written into labour contracts ex-ante.

the endogenous response of heterogeneous firms will help to deepen our understanding of how firms, industries and economies respond to policy reforms such as deregulation. For simplicity and for clarity of exposition we assume that wages are determined as

$$(23) \quad w = 1 + (r + \delta)(EFC),$$

where  $EFC$  denotes effective firing costs depicted in  $= p_f + \sqrt{2 \left( c_{f_1} + \frac{c_{f_2}}{(1 + e^{-c_0(L-l)})} \right) \gamma_f}$ . Equation (23) constitutes an additional “wage mechanism” in the regulation transmission channel and can be seen as encompassing various sources of wage rigidity. While being a short cut to a strictly micro founded wage equation, equation (23) constitutes a plausible starting point for analysing the impact of endogenous wages on the regulation transmission process. Figure 5 reports the numerical results of this experiment for  $A_0 = 0.08$ ,  $A_0 = 0.16$ , and  $A_0 = 0.24$ , respectively.

**Figure 5: Hiring and Firing Thresholds with Endogenous Wages for  $l = 10$**



Adjustment costs of labour again induce firms to hoard labour during recessions, and also to hire fewer workers during boom periods. Contrary to the baseline model, however, the shape of the firing cost profiles implies that the insider-outsider considerations provide a channel through which the impact of firing costs is pulled, via wages, towards a negative impact on average employment.<sup>18</sup>

In this Section we have provided some tests of the sensitivity and robustness of the baseline model. Taken together, the augmented model follows a more pronounced deregulatory line. Of course, the endogenous productivity and wage effects are hard to quantify. Nevertheless, the paper shows that they are important channels through which employment protection might affect macroeconomic aggregates and therefore they are the ones that should be the focus of attention.

<sup>18</sup> This result confirms the “all or nothing” warning issued by Coe and Snower (1997) and Orszag and Snower (1998). They argue that piecemeal labour market reforms may have had so little success because they disregarded the complementarities between a broad range of policies and institutions. The findings of both papers are drawn from distinctly different frameworks than our modelling approach. The parallels with our findings are nonetheless intriguing.

### 3. Temporary Work Agencies (*Zeitarbeit*)

Next, we consider other forms of employment which increase the flexibility of the labour market. In particular, we are modelling the contribution of temporary employment agencies. This is still a blank cell in the real options modelling literature.

Temporary agency work can broadly be defined as a temporary employment relationship between a temporary work agency, which is the employer, and a worker, where the latter is assigned to work for, and under the control of a firm making use of his/her services. In order to provide the background of flexible employment, some information about the institutional settings in Germany is provided in the next paragraph.<sup>19</sup>

The number of workers employed in temporary employment agencies (*Zeitarbeiter*) has increased in Germany as a consequence of deregulation of this sector in the 1990s.<sup>20</sup> Already in August 1972, the *Bundestag* passed the Act on Temporary Employment Business (*Arbeitnehmerüberlassungsgesetz*). In 1985, the Recruitment Promotion Act (*Beschäftigungsförderungsgesetz*) prolonged the maximum ‘hiring out’-period to one firm from three to six months. In January 1994, the maximum ‘hiring out’-period was extended to nine months. The posted work period was then extended stepwise to 12 months in 1997, to 2 years in 2002, and finally the limit was completely abolished in 2003 – except the ban of temporary employment agencies in the construction sector. Since then there are no legal limitations to signing overlapping temporary contracts for any worker at the same firm. The labour market reform therefore brought about the creation of a dual contract system in Germany in which temporary workers create a buffer peripheral workforce as a hedge against market uncertainty.

How can we model the atypical form of *Zeitarbeit* enabling quick (external) employment adjustments in client firms?<sup>21</sup> In the context of uncertainty about the state of the economy in the future, the firm faces the problem of deciding the optimal number of workers, where there are two types of labour contracts: standard open-ended contracts and *Zeitarbeit* contracts. In order to model the interplay between external and internal work force adjustments, we adopt the Cobb-Douglas production function with constant returns to scale and two types of workers

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<sup>19</sup> German labour law defines the hiring-out of labour as a legal relationship where a business lends temporarily an employee with whom it had concluded a permanent employment contract to another employer while the legal relationship with the first one continues to apply and the employee is obliged to work for the company and in line with the instructions of the hiring-out employer. Because the employment agency remains the employer of temporary agency workers, social benefits carry over from one assignment to another. In addition, relevant labour laws (social security schemes, paid holidays, maternity leave) fully apply to temporary agency work.

<sup>20</sup> The up-to-date BMW factory in Leipzig is an example. In all, 5,200 workers are in the plant of whom 1,300 are *Zeitarbeiter* hired from an agency. Again we use Germany as an example but the German experience isn't unique. Several other countries have similar and in some cases even more of those types of employment. In the EU about 7 million workers are currently employed by private employment agencies. Neugart and Storrie (2006) show that the improvement in the matching technology of temporary work agencies may lead to the emergence and growth of temporary work.

<sup>21</sup> An alternative to external forms of flexibility is provided by internal working time flexibility (overtime, short time work). For an analysis of working time flexibility in a real options framework, see Chen and Funke (2004).

$$(24) \quad Y_t = AK^\alpha L_t^{1-\alpha-\gamma} N_t^\gamma, \quad 0 < \alpha, \gamma < 1,$$

where  $N$  represents the number of temporary employees and  $\gamma$  denotes the share of temporary employees in the production function.<sup>22</sup> Combined with equation (2), we have the following immediate profit function,

$$(25) \quad \Pi = A^{1/\psi} Z_t K^{\alpha_1} L_t^{\alpha_2} N_t^{\alpha_3} - wL_t - w_N N_t - C(M_t, L_t),$$

where  $w_N$  is the wages for temporary employees,  $\alpha_1 = \alpha/\psi$ ,  $\alpha_2 = (1 - \alpha - \gamma)/\psi$ , and  $\alpha_3 = \gamma/\psi$ . *Zeitarbeiter* get wages of around 75 percent of permanent workers on average but the cost to the hiring company is much more than that because it must pay the agency its cut on top of the wage that goes to the workers, i.e. we have  $w < w_N$ .

One motivation for temporary hirings is that they can be terminated at the end of their term at low cost or no cost at all. It is therefore assumed that there are no hiring and/or firing costs for  $N$ . Thus,  $C(\cdot)$  is not a function of  $N$  and we do not need to consider the real options term for  $N$  due to no sunk costs from hiring/firing *Zeitarbeit* employees. The firms' optimal temporary and permanent employment levels are obtained maximising the expected discounted value of the firms' future cash flows. The first order condition for  $N$  is a function of  $Z$  and  $L$ :

$$(26) \quad \alpha_3 A^{1/\psi} Z K^{\alpha_1} L^{\alpha_2} N^{\alpha_3-1} = w_N \Rightarrow N = \left[ \frac{w_N}{ZA^{1/\psi} \alpha_3 K^{\alpha_1} L^{\alpha_2}} \right]^{\frac{1}{\alpha_3-1}}.$$

As  $Z$  moves to hiring thresholds, the firm increases its level of  $N$  and raise the hiring thresholds above the ones of the case without temporary employment. Similarly, as  $Z$  falls, the firm initially reduces the number of temporary workers which dampens the need to fire permanent workers. The effect of introducing temporary workers indeed serves as buffer and widens the inaction area. The discussion means that before the firm takes any actions of hiring and firing, it first and continuously adjust its level of  $N$  to optimise profits. In other words, the optimal  $N$  for hiring and firing decisions are considered as constants since it is determined first before the decisions of hiring and firing. Therefore, the value-matching conditions of hiring and firing of equations (18) and (19) become

$$(27) \quad \frac{\alpha_2 A^{1/\psi} Z_H K^{\alpha_1} L^{\alpha_2-1} N_H^{\alpha_3}}{r + \alpha_2 \delta - \eta} - \frac{w}{r + \delta} + B_2 \left( Z_H K^{\alpha_1} L^{\alpha_2-1} \right)^{\beta_2} = p_h + \sqrt{2c_h \gamma_h} + B_1 \left( Z_H K^{\alpha_1} L^{\alpha_2-1} \right)^{\beta_1},$$

and

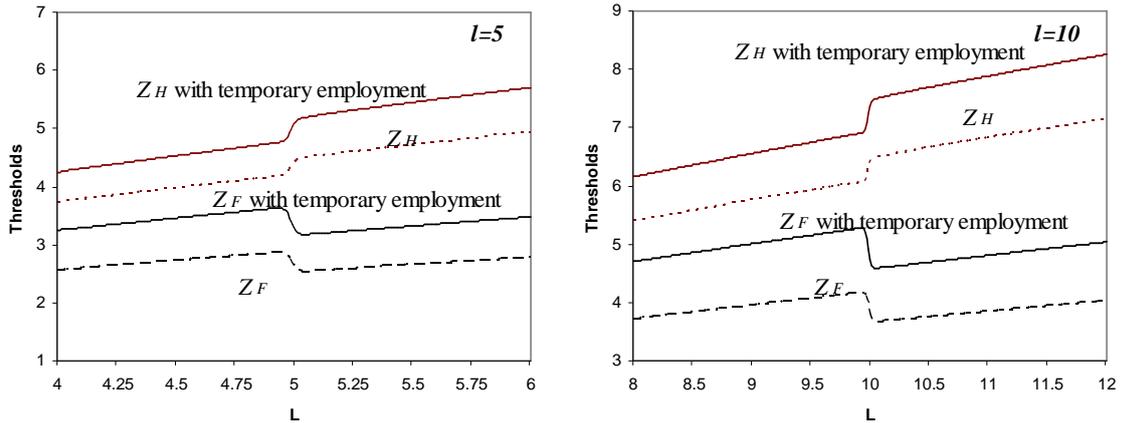
$$(28) \quad - \left[ \frac{\alpha_2 A^{1/\psi} Z_F K^{\alpha_1} L^{\alpha_2-1} N_F^{\alpha_3}}{r + \alpha_2 \delta - \eta} - \frac{w}{r + \delta} \right] + B_1 \left( Z_F K^{\alpha_1} L^{\alpha_2-1} \right)^{\beta_1} \\ = p_f + \sqrt{2 \left( c_{f_1} + \left( \frac{c_{f_2}}{1 + e^{-c_0(L-l)}} \right) \right) \gamma_f} + B_2 \left( Z_F K^{\alpha_1} L^{\alpha_2-1} \right)^{\beta_2},$$

where  $N_H$  and  $N_F$  are determined by  $N_{H/F} = \left[ \frac{w_N}{Z_{H/F} A^{1/\psi} \alpha_3 K^{\alpha_1} L^{\alpha_2}} \right]^{\frac{1}{\alpha_3-1}}$ . The smooth-pasting

conditions follows by differentiating equations (30) and (31) with respect to hiring and firing thresholds respectively.

We are now able to say something about the relationship between employment protection thresholds and *Zeitarbeit*. Using available evidence for Germany on wages for temporary employees and on the average temporary work to regular work ratio, our calibrated parameters are  $w_N = 1.4w$  and  $\gamma = 0.05$ .

**Figure 6: The Effects of Dismissal Protection Regulation with *Zeitarbeit* and Exempted Establishment  $L < 5$  vs.  $L < 10$**



**Note:** The solid lines give the no action areas for the case with temporary employment (*Zeitarbeit*); the dashed lines provide the baseline framework without temporary work (see Figure 2 and 3).

<sup>22</sup> In the setup we therefore assume that both categories of workers are close substitutes. For the sake of realism one might argue that a CES function might be more appropriate. Chen and Funke (2004, 2007) have demonstrated that replacing the CD by a CES production function doesn't change any qualitative conclusion.

Focusing again on the thresholds, the calibration results in Figure 6 indicate that *Zeitarbeit* leads to higher hiring and firing thresholds for permanent workers across all firm sizes. Thus, temporary workers create a buffer peripheral workforce as a hedge against market uncertainty. This notwithstanding, the effects of employment protection are different across firm size. Ocular inspection indicates that the widening of the no action areas at the thresholds  $l = 5$  vs.  $l = 10$  is more pronounced in case of *Zeitarbeit* being available. In other words, the results support the prediction that more extensive employment protection mandates for standard employment contracts increases the incentives for firms to hire temporary workers. Therefore, the main conclusion of the previous analysis is that *Zeitarbeit* is positively correlated with permanent employment protection.

#### 4. The Cyclicity and Dynamics of Employment

Since the focus of the paper is employment, we next present a translation from thresholds to employment and assess the impact of alternative thresholds upon  $L$ . In order to get a clear “feel” for the dynamics of the model, we first have to specify a solution method that will lead us to generate discrete realizations of the level of employment, given the chosen levels of parameters. Several options are available at this point, but the structure of the model readily suggests using a sequential iterations method. It works as follows. Equation (6) is proxied by the following discrete stochastic differential equation - the Euler scheme,

$$(29) \quad Z_{t+1} = Z_t + \eta Z_t \Delta t + \sigma Z_t \varepsilon_t \sqrt{\Delta t}, \quad \varepsilon_t \sim N(0;1),$$

where the normal random variables,  $\varepsilon_t$ , are generated via the central limit theorem and the Box-Muller (1958) method for transforming a uniformly distributed random variables to a normal distribution with given mean and variance and  $\Delta t$  represents small changes in  $t$ .

As long as the discrete time series values for  $Z_t$  wander within the inaction area, the firm does nothing – no hires or fires – except for natural attrition of employees due to quits. Once  $Z_t$  hits the hiring thresholds, the firm then hires  $M\Delta t = \sqrt{2c_h/\gamma_h} \Delta t$  employees according to equation (10) for each step of  $\Delta t$ , and the changes of employees are governed and proxied by the following equation,

$$(30) \quad L_{t+1} = L_t + \left[ \sqrt{\frac{2c_h}{\gamma_h}} - \delta L_t \right] \Delta t,$$

which is discrete-time version of equation (5) with  $M = \sqrt{2c_h/\gamma_h}$ . Similarly, if  $Z_t$  hits the firing thresholds, the firm is then starting to fire  $\sqrt{2(c_{f_1} + c_{f_2}/(1 + e^{-c_0(L-l)}))}/\gamma_f \Delta t$  [from equation (11)], and the process of changes of employees is denoted by the following equation,

$$(31) \quad L_{t+1} = L_t + \left[ -\sqrt{2\left(c_{f_1} + \frac{c_{f_2}}{(1 + e^{-c_0(L-l)})}\right)}/\gamma_f - \delta L_t \right] \Delta t, \quad L_t > 0.$$

Note that equations (30) and (31) are approximate equations since beyond the thresholds – outside of the inaction regime, the hires and fires for period  $\Delta t$  are bound to be greater than  $\sqrt{2c_h/\gamma_h} \Delta t$  and  $\sqrt{2(c_{f_1} + c_{f_2}/(1 + e^{-c_0(L-l)}))}/\gamma_f \Delta t$ , respectively. Note that the thresholds for hiring and firing are changing at every time step since  $L_t$  changes over time due to quits as well as possible hiring or firing.

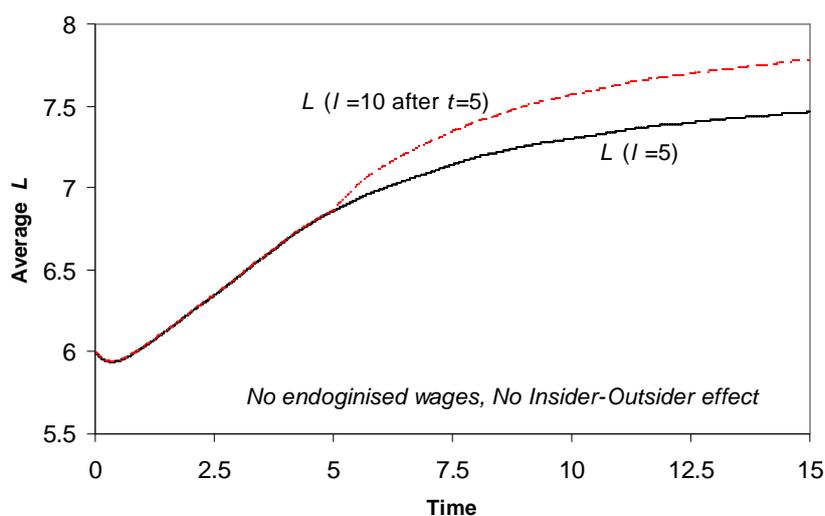
We first concentrate upon the benchmark model without *Zeitarbeit*. Figures 7 and 8 demonstrate the dynamic effect of an unanticipated change in  $l$  upon the employment level over time. The average path of employment in each Figure is averaged from 5000 Monte Carlo simulations over a 15 year period with  $\Delta t = \dots$ ,  $Z_0 = \dots$  and initial value of  $l = 5$ .<sup>23</sup> All other parameters are the same as the benchmark values except the unanticipated change from  $l = 5$  to  $l = 10$  at  $t = 5$ . Figure 7 shows the dynamic impact of this policy change for exogenous wages ( $w = 1$ ), while Figure 8 focuses on the effect with endogenous wages and  $A_0 = 0.24$ .<sup>24</sup>

The reforms can be seen as labour markets which have been partly freed up, to capture what has been the German experience. The graphs suggest that employment increases on average after the policy change, i.e. firms hire more and fire less once the regulation is relaxed at  $t = 5$ . The comparison of Figure 7 and 8 reveals that the increase in employment is larger for the scenario with endogenous wages. The underlying reason is that the new regime ( $l = 10$ ) reduces the rigidities in the wage-bargaining process. Insiders feel less insulated and therefore demand lower wages in the awareness that any resultant job losses will also fall on insiders.

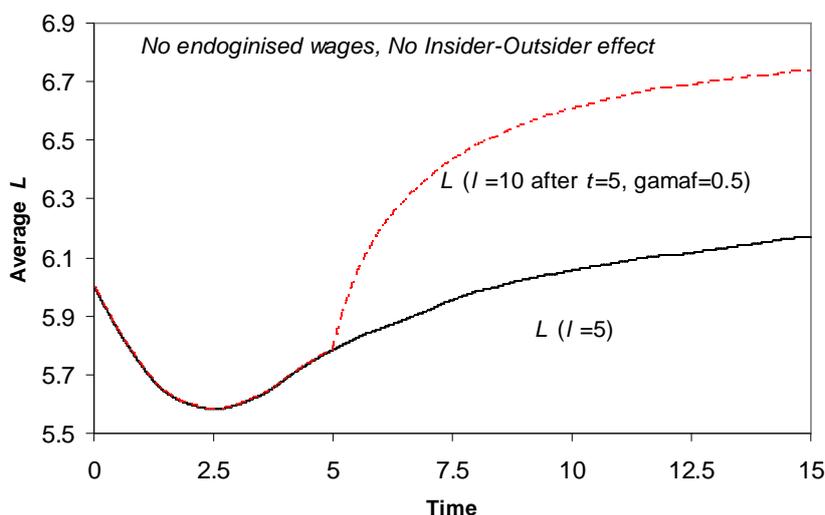
<sup>23</sup> Experimentation with larger numbers of runs shows no significant change to the results.

<sup>24</sup> We ignore behavioural assumptions regarding market rivalry, which in turn would necessitate some kind of game-theoretic analysis to take account of the strategic interactions among the firms, results of which are in turn heavily dependent on assumptions regarding the information sets available and the type of game being played. The ramifications of competitive interaction on the decision making of firms have been discussed by Leahy (1993). Leahy (1993) has shown that the assumption of myopic firms who ignore the impact of other firms' actions results in the same critical boundaries that trigger factor demand as a model in which firms correctly anticipate the strategies of other firms. Grenadier (2002) has recently extended Leahy's (1993) "Principle of Optimality of Myopic Behavior" to the apparently more complex case of dynamic oligopoly under uncertainty. Both papers therefore permit to bypass strategic general equilibrium considerations when analysing factor demand under uncertainty. An alternative way forward is the general equilibrium version of the Bentolila-Bertola model solved by Hopenhavn and Rogerson (1993).

**Figure 7: The Effect of Changes in  $l$  on Average Employment  $L$  with Exogenous Wages**



**Figure 8: The Effect of Changes in  $l$  on Average Employment  $L$  with Endogenous Wages**

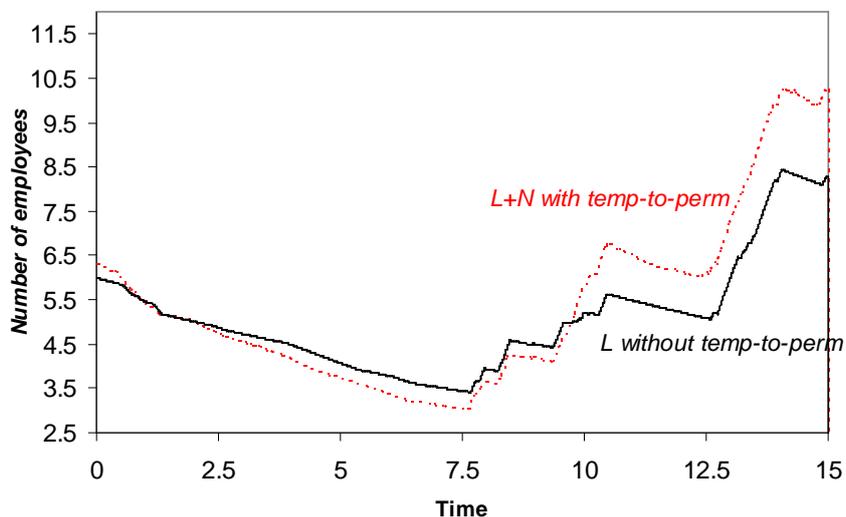


Finally, we will analyse the cyclicity of temporary agency work. How do the two types of labour input respond to shocks? Figure 9 demonstrates the dynamic effect of *Zeitarbeit* upon employment, using the previous algorithm with constant  $w = 1$ ,  $w_N = 1.4w$ , and  $l = 5$ . When comparing the bold line (only permanent employment contracts) with the dashed line (permanent employment contracts and temporary agency work), it can be seen that permanent employment and *Zeitarbeit* exhibit strikingly different cyclical behaviour with *Zeitarbeit* being more volatile.<sup>25</sup> Employers use temporary agency work to overcome short-term demand peaks instead of hiring workers on normal employment contracts, i.e. firms use the peripheral workforce as a buffer. An important conclusion is that the ratio of temporary to permanent workers increases after positive shocks and decreases after negative shocks.

<sup>25</sup> This is consistent with Messina and Vallanti (2005) showing that more stringent firing restrictions dampen the response of job destruction and job creation to business cycles in 14 European countries.

The implication is that the peripheral workforce is more strongly exposed to shocks while the job security of the (smaller) core workforce is tightened. In this spirit, temporary employment agencies facilitate flexibility and the core-periphery hypothesis is supported.

**Figure 9: The Effect of *Zeitarbeit* on the Dynamic Path of Employment with  $\gamma = 0.05$  and  $\alpha = 0.65$**



## 5. Conclusions

Attitudes and policies towards deregulation of labour markets have been subject to considerable controversy and flux. Our paper fits neatly into this debate, and provides some fresh evidence on labour demand dynamics associated with asymmetric job security provisions across the firm size distribution. There has been considerable debate among politicians, unions, employer associations and economists about dismissal protection legislation. To contribute to this debate, we have designed and presented an economically meaningful and transparent dynamic model in continuous time characterizing the firm's optimal behaviour under uncertainty. While highly stylised, the real options model singles out important transmission channels and allows policy-makers to study the implications of policy interventions in alternative model specifications. We have analysed the impact of employment protection legislation on the incidence of regular and temporary hirings, firings, and employment. We have argued on theoretical and numerical grounds that protection will tend to lower the incidence of regular contracts and increases the input of *Zeitarbeit*.

## Appendix A: The Boundaries of the Inaction Area

By substituting (9) in the text back into the Bellman equation (8) in the text and rearranging we obtain for the hiring and firing decisions:

$$(A1) \quad rV = A^{1/\psi} ZK^{\alpha_1} L^{\alpha_2} - wL - c_h + \frac{1}{2} \frac{(v - p_h)^2}{\gamma_h} - \delta vL + \eta ZV_Z + \frac{1}{2} \sigma^2 Z^2 V_{ZZ};$$

$$(A2) \quad rV = A^{1/\psi} ZK^{\alpha_1} L^{\alpha_2} - wL - c_{f_1} - \frac{c_{f_2}}{1 + e^{-c_0(L-l)}} + \frac{1}{2} \frac{(v + p_f)^2}{\gamma_f} - \delta vL + \eta ZV_Z + \frac{1}{2} \sigma^2 Z^2 V_{ZZ}.$$

The firm would hire/hire marginal employees only if the total revenue net costs of hiring/hiring are non-negative. Thus, for hiring decision ( $M \geq 0$ ), the firm has benefit of hiring  $M$  employees – the value of the firm increases by  $Mv$ ; for hiring those  $M$  employees, the firm pays the total cost of employment for hiring. The hiring decisions would only happen for a certain  $M$  or greater as long as the following equation is satisfied:

$$(A3) \quad Mv - \left( c_h + p_h M + \frac{1}{2} \gamma_h M^2 \right) \geq 0.$$

In economic downturns, the firm endures a loss so that the value of  $v$  is negative. By firing  $|M|$  employees ( $M \leq 0$ ), the loss of the firm is reduced by  $Mv$ , which is considered to be the benefit of firing  $|M|$  employees; the firing also incurs some total cost of adjustment. The firm only fire a certain number of employees or more if the following relationship is satisfied:

$$(A4) \quad Mv - \left( c_{f_1} + \frac{c_{f_2}}{1 + e^{-c_0(L-l)}} - p_f M + \frac{1}{2} \gamma_f M^2 \right) \geq 0.$$

Multiplying both sides of (9) in the text by  $M$  and substituting into (A3) and (A4) gives

$$M^2 \geq \frac{2c_h}{\gamma_h} \text{ for hiring,}$$

and

$$M^2 \geq 2 \left( c_{f_1} + \frac{c_{f_2}}{1 + e^{-c_0(L-l)}} \right) / \gamma_f \text{ for firing.}$$

Thus, for (mass-) hiring starting thresholds, we shall have

$$(A5) \quad M \geq \sqrt{\frac{2c_h}{\gamma_h}} > 0;$$

and for (mass-) firing starting thresholds, we need the following relationship

$$(A6) \quad M \leq -\sqrt{2 \left( c_{f_1} + \frac{c_{f_2}}{1 + e^{-c_0(L-l)}} \right) / \gamma_f} < 0.$$

Substituting (A5) and (A6) back into equation (9) in the text respectively gives the hiring/firing regimes for the intertemporal marginal value of the firm

$$v \geq p_h + \sqrt{2c_h\gamma_h} \quad \text{for hiring regime,}$$

and

$$v \leq -p_f - \sqrt{2\left(c_{f1} + \frac{c_{f2}}{(1 + e^{-c_0(L-l)})}\right)\gamma_f} \quad \text{for firing regime.}$$

The boundaries of the inaction area or the beginning points of hiring and firing regimes, where equations hold, are then determined by the following two equations.

$$(A7) \quad v = p_h + \sqrt{2c_h\gamma_h} \quad \text{for hiring thresholds,}$$

and

$$(A8) \quad v = -p_f - \sqrt{2\left(c_{f1} + \frac{c_{f2}}{(1 + e^{-c_0(L-l)})}\right)\gamma_f} \quad \text{for firing thresholds.}$$

Substituting (A7) and (A8) back into Bellman equations (A1) and (A2) gives the following unified differential equations for hiring and firing:

$$(A9) \quad rV = A^{1/\psi} ZK^{\alpha_1} L^{\alpha_2} - wL_t - \delta vL + \eta ZV_Z + \frac{1}{2} \sigma^2 Z^2 V_{ZZ}.$$

Using the definitions  $v = V_L$ ,  $v_Z = V_{LZ}$ ,  $v_L = V_{LL}$  and  $v_{ZZ} = V_{LZZ}$  and differentiating both sides of equation (A9) with respect to  $L$  yields

$$(A10) \quad (r + \delta)v = \alpha_2 A^{1/\psi} ZK^{\alpha_1} L^{\alpha_2-1} - w - \delta v_L L + \eta Zv_Z + \frac{1}{2} \sigma^2 Z^2 v_{ZZ},$$

which is equation (14) in the text.

## Appendix B: The Particular and General Solutions for $v$

### Particular solutions

Assume that the particular solutions have the following functional form:

$$(B1) \quad v^P = aZK^{\alpha_1} L^{\alpha_2-1} + b,$$

We then have the following relationships:

$$(B2) \quad \eta Zv_Z = a\eta ZK^{\alpha_1} L^{\alpha_2-1},$$

$$(B3) \quad v_{ZZ} = 0,$$

$$(B4) \quad -\delta L v_L = -a \delta (\alpha_2 - 1) Z K^{\alpha_1} L^{\alpha_2 - 1}.$$

Substituting into equation (14) in the text gives:

$$(B5) \quad \left[ a(r + \alpha_2 \delta - \eta) - \alpha_2 A^{1/\psi} \right] Z K^{\alpha_1} L^{\alpha_2 - 1} + [(r + \delta)b + w] = 0.$$

The above equation should hold for any value of marginal product of employees. Thus, we have

$$(B6) \quad a = \frac{\alpha_2 A^{1/\psi}}{r + \alpha_2 \delta - \eta},$$

$$(B7) \quad b = -\frac{w}{r + \delta},$$

which yields the particular solution (15) in the text.

### *Homogenous solutions*

The homogenous part of equation (14) in the text is represented by

$$(B6) \quad (r + \delta)v = -\delta v_L L + \eta Z v_Z + \frac{1}{2} \sigma^2 Z^2 v_{ZZ}.$$

The homogenous solutions should have the same components as in particular solutions. Therefore, assume the following functional form for homogenous solutions:

$$(B7) \quad v^H = B \left( Z K^{\alpha_1} L^{\alpha_2 - 1} \right)^\beta,$$

where  $A$  is constant and to be determined by value-matching and smooth-pasting conditions. We then have the following relationships for homogenous solutions:

$$(B8) \quad \eta Z v_Z = \eta \beta B \left( Z K^{\alpha_1} L^{\alpha_2 - 1} \right)^\beta,$$

$$(B9) \quad \frac{1}{2} \sigma^2 Z^2 v_{ZZ} = \frac{1}{2} \sigma^2 \beta (\beta - 1) B \left( Z K^{\alpha_1} L^{\alpha_2 - 1} \right)^\beta,$$

$$(B10) \quad -\delta L v_L = -\delta (\alpha_2 - 1) \beta B \left( Z K^{\alpha_1} L^{\alpha_2 - 1} \right)^\beta.$$

Substituting into equation (B6) and rearranging gives:

$$(B11) \quad r + \delta + \delta \beta (\alpha_2 - 1) - \eta \beta - \frac{1}{2} \sigma^2 \beta (\beta - 1) = 0,$$

which is (17) in the text. There are two characteristic roots for  $\beta$ : one positive and one negative:  $\beta_1 > 0 > \beta_2$ . Therefore, the homogenous (general) solutions are shown as follows:

$$(B12) \quad v^H = -B_1 \left( ZK^{\alpha_1} L^{\alpha_2-1} \right)^{\beta_1} + B_2 \left( ZK^{\alpha_1} L^{\alpha_2-1} \right)^{\beta_2},$$

which corresponds to real options to hire and fire employees respectively, and is equation (16) in the text.

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