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Life-Cycle, Effort and Academic Inactivity

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Abstract

It has been observed that university professors sometimes become less research active in their later years. This paper models the decision to become inactive as a utility maximising problem under conditions of uncertainty and derives an age-dependent activity condition for the level of research productivity. The model implies that professors who are close to retirement age are more likely to become inactive when faced with setbacks in their research while those who continue research do not lower their activity levels. Using data from the University of Iceland, we find support for the model’s predictions. The model suggests that universities should induce their older faculty to remain research active by striving to make their research more productive and enjoyable, maintaining peer pressure, reducing job security and offering higher performance related pay.

Keywords: Inactivity, aging, optimal stopping.
JEL Classification: J44, J22

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1. Introduction

Young university lecturers frequently complain about colleagues who are not engaged in research, who have become “deadwood” in common parlance. However, the reasons why university teachers may end up being inactive are seldom explained. Workers in other professions may also gradually reduce their effort as they get older. Some workers may also choose to move to professions where opportunities for on-the-job leisure are greater: Politicians sometimes end up as diplomats; football players as celebrities and movie stars may take on fewer roles and end up enjoying leisure and sometimes fame. In some cases the decision is driven by physical deterioration, such as in sports, but in other cases it is for other less well-defined reasons such as when an academic stops spending his time doing research.\(^1\)

The question that then arises is whether this is due to declining mental abilities or results from changing incentives. The objective of this paper is to show how older intertemporal-utility-maximising workers may face incentives to stop doing research in spite of undiminished physical and mental strength.

We will explain our argument using the university workplace as an example but it applies also to other professions where workers’ level of exertion is at least partly up to their own discretion and can only be imperfectly monitored by employers. Our model shows why academics may face reduced incentives to do research as they get closer to retirement. One insight coming from our model is that older workers are less threatened by the possibility of being dismissed since they have fewer years left in the labour market and future research successes are also less important for them when compared to current sacrifices brought about by strenuous research effort.\(^2\) The model also helps distinguish those individuals who are more likely to become inactive. Those who remain research active are the ones who enjoy research and those whose productivity is sufficiently great to offset any incentives to slow

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\(^1\) Even in the case of athletes, the retirement decision is to some extent up to the individual’s discretion because the rate of deterioration of physical ability has been shown to be quite small. See Fair (1994, 2007) who fails to find a strong effect of aging on physical abilities.

\(^2\) The model resembles the model of Lazear and Rosen (1981) on rank-order tournaments. In their model workers exert effort in the hope of being promoted in the future while in our model they exert effort in the hope of enjoying an unexpected productivity surge.
down. It follows that while there are many inactive older professors, the active professors tend to be quite productive.

2. Causes of inactivity in late career

To our knowledge, not much has been written on the causes of inactivity in late career. There is however statistical evidence showing that research productivity is declining in age. Oster and Hamermesh (1998) find that economists’ productivity measured by publications in leading journals declines with age, although the probability of acceptance, once an article has been submitted to a leading journal, is independent of age. Moreover they find that the median age of authors of articles in leading economics journals was 36 in the 1980s and the 1990s and that a very small minority of authors are over 50 in spite of a substantial percentage of AEA members being over the age of 50. However, they cannot discriminate between the two possible reasons for this observation; whether the falling frequency of publications is due to deteriorating mental faculties or, alternatively, reflects rational decisions to devote less time to research. In a recent paper, Jones (2010) analyses the age of individuals at the time of their greatest achievements in science using data on research that leads to the Nobel Prize in physics, chemistry, medicine and economics and also data on research that leads to great technological achievements as shown in the almanacs of the history of technology. He finds that the greatest concentration of innovations in the life of a scientist comes in his 30s but a substantial amount also comes in the 40s, while scientists in their 50s, and even more so in the 60, generate far fewer discoveries.

Standard human capital models can be used to explain why research effort may be declining in age. Using the Mincer (1958) framework, workers have to choose between working to earn wages and investing in human capital. In the Ben-Porath (1967) model, we can think of "academic human capital" as having a particularly high depreciation rate – a lot of work is required just to keep up to date – and that as people get close to retirement they have a smaller incentive to invest in human capital since there are few years left to reap the return from this investment. The empirical prediction of the human capital approach is that

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3 Similar results are reached by Lehman (1953), Diamond (1986), McDowell (1982) and Levin and Stephan (1992) for other disciplines. In contrast, Jan van Ours (2009) finds no relationship between the quality-adjusted rate of publication and age among his colleagues at Tilburg University.
human capital and research activity is declining in age. We will refer to this as the “human capital effect.”

In our model output is partly stochastic – sometimes an editor or a referee likes a paper because he happens to be working on something similar; clearly the choice of referees has a random element; co-authors may be active or not; and we have family issues that can distract us and so forth. Faced with disappointments, the young may decide to stay research active because they can hope for future successes knowing that they have a long future ahead. In contrast, the older professors may think that they are not likely to be lucky in the short remaining span of their tenure and become inactive. Remaining research active allows a professor to hope for future (publication) success, the more years he has left the more time he has to realize these hopes. Hence a young lecturer may stay research active in spite of setbacks while the older one may turn to admin/teaching/consulting. It follows that the old professors are either very good or inactive while the mediocre younger ones decide to stay research active in the hope of future successes. We will refer to this as the “option value effect.”

3. A Model of Academic Inactivity

In this section we model the decision by a professor whether to remain research active. The representative professor devotes his time to teaching, administration and research. Of these tasks, his efforts at teaching and administration are observable by the university and justify paying him a fixed salary $w_0$. In contrast, there is asymmetric information about research effort. Low observable output in the form of published papers and books can have many possible explanations, such as excessive attention to detail by the professor, the research projects being very ambitious and time consuming, bad luck when it comes to the choice of journals and publishers for submitting research results or simply that the professor is engaged in the type of research that the profession does not value at the moment due to fads and fashions. There is also, of course the possibility that the professor is simply not devoting enough time to doing research.
We first explore the case when it is impossible to monitor research effort but then extend the analysis in Section 4 to the case of imperfect monitoring of effort and finally to include tenure effects in Section 5. Empirical predictions are presented in Section 6 before the concluding Section 7.

3.1 Assumptions

We assume that a representative professor faces a one-off decision whether or not to continue doing research in the future. The level of effort, \( f \), has two possible values, zero and one as in Shapiro and Stiglitz (1984); when \( f \) is equal to one the professor is doing research while \( f \) takes the value zero when he is inactive in research although still teaching and performing administrative duties. When doing research the professor suffers disutility \( \gamma \) – caused by the constant exertion needed to get results – but generates measurable research output \( g \) which the university uses to calculate his performance-related pay. The variable \( \gamma \) can take a negative value if the professor enjoys doing research – in which case he will never choose to become inactive. When a professor decides to become inactive in research, that is \( f \) becomes zero, he faces the one-off wrath of his colleagues which we measure by the variable \( W \).4 However, continuing research effort gives an uncertain return in the form of research output that is used to calculate performance-related pay.

The professor’s pay is a linear function of observable research output \( g \);

\[
(1) \quad w = w_0 + w_bg, f, \quad w_0, w_b > 0
\]

It follows that a worker not doing research would receive the basic professorial pay \( w_0 \) and a worker engaged in research would receive \( w_0 + w_bg \), while having disutility \( \gamma \) from doing the work.5

Future research output is uncertain because of the possibility that research effort will not generate sufficiently interesting and innovative research results, because of uncertainty about

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4 One can also model part of the reaction of colleagues as a constant expression of disapproval when not doing research. In this case one can define the variable \( \gamma \) as the disutility of effort net of the approval of colleagues when doing research.

5 The model can be easily modified to take into account wage compression – differences in productivity between two workers exceeding differences in wages – as in Frank (1984) by raising \( g \) in equation (1) to a power which is less than one. See also Booth and Zoega (2004) on the effect of wage compression for human capital investment.
how quickly results will be achieved and also because of uncertainty about the reception by editors of professional journals whose preferences are difficult to predict. Each individual can also expect his productivity to change in the future, depending on the environment in which he finds himself, the extent of learning by doing, personal circumstances, the productivity of colleagues and collaborators and so forth. To capture these dynamics, it is assumed that the level of $g$ follows a geometric drifted Brownian motion,

$$dg = \eta g dt + \sigma g dz,$$

where $\eta$ is the drift parameter of research output, $\sigma$ is the uncertainty parameter about future research output and $z$ is the standard Wiener process – a normalised Gaussian process with independent increments.

The professor has to make up his mind whether to do research or not, keeping in mind that although not doing research yields utility in the form of leisure at work, it also reduces the level of research output and hence also the amount of performance-related pay. The professor thus faces an optimal stopping problem when he decides whether to shirk his research duties. We make the assumption that the professor cannot resume researching once he has decided to shirk.  

3.2 The research decision

The professor has utility which is linear in wages and the disutility of doing research; $w_0 + w_s g_t - \gamma$ when doing research and $w_0$ when not doing research. This gives the following intertemporal utility function;

$$V = E \left[ \int_t^T (w_0 + w_s g_s f - \gamma f) e^{-\rho (t-s)} ds \right],$$

subject to (2), where $E[ \ ]$ is the expectation operator, $\rho$ is the discount rate of the professor and $T - t$ is the time remaining until the professor retires. If the professor chooses not to do research, the intertemporal utility for not being research active $V^5$ is

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6 Implicitly, it means that there are large sunk costs involved in resuming research such that the option to resume research approaches null value for the decision not to do research; for example, the laboratory is gone forever or the human capital has depreciated.
which is obtained by integrating the integral directly as there is no way back to research in the future. In contrast, if he chooses to do research the professor has the following intertemporal utility $V^R$

$$V^R = E \left[ \int_T^T (w_0 + w_b g_s - \gamma) e^{-\rho(t-s)} dt \right],$$

where the difference between (4) and (5) lies in the research-active professor expecting performance-related pay but also enduring the disutility of more effort $\gamma$, which can be either positive or negative, negative if the professor enjoys the effort and challenges of research effort, otherwise positive.

The decision by the professor whether to discontinue research depends on whether the discounted utility from not being research active for the remainder of his tenure $V^S$ exceeds the sum of the discounted utility from being research active $V^R$ and the expected discomfort from the wrath of colleagues when stopping research $W$. The equality of the two generates what we call the **activity condition**

$$V^S = V^R + W.$$

Equation (6) is analogous to the non-shirking condition of Shapiro and Stiglitz (1984). It captures both the *human capital effect* described by Ben-Porath (1967) and the *option value effect*, described earlier. The condition determines the research productivity level $g$ – hence the wages $w_0 + w_b g$ – that is needed to convince the professor to continue doing research for a given performance-related pay $w_b$, a system of measuring performance $g$, the disutility from doing research $\gamma$, peer pressure $W$ and, as we will show, age.

### 3.3 The activity condition

We need to solve the activity condition. While the solution for $V^S$ is given by equation (4), we still need to solve for $V^R$. The Bellman equation for equation (5) is the following,

$$\rho V^R = w_0 + w_b g - \gamma + \eta g V^R_g + \frac{1}{2} \sigma^2 g^2 V^R_{gg} + V^R_t,$$
where \( w_0 + w_0g - \gamma \) represents the net utility from working at the university, \( \eta g V^g \) shows changes in \( V^R \) due to a drift in research productivity, and the last two terms denote changes in \( V^R \) due to diffusion.

The solution to equation (7) comprises a particular solution, representing the net benefits from doing research for the rest of one’s career – the human capital effect of Ben-Porath – and a homogenous solution, which is equivalent to the value of the real option to discontinue research later. Therefore, we have the following solutions for \( V^R \) (see Appendix I for details),

\[
V^R = \left( \frac{w_0 - \gamma}{\rho} \right) (1 - e^{-\rho(T-t)}) + \frac{w_0g}{\rho - \eta} (1 - e^{-(\rho-\eta)(T-t)}) + A_2 g^\beta_2 N(-d_2),
\]

where the first two terms of the right-hand side are obtained from directly integrating the intertemporal utility; the last term on the right-hand side denotes the real option to discontinue research and \( A_2 \) is an unknown parameter to be determined by the value-matching condition of the optimal stopping problem.\(^7\) The parameter \( \beta_2 \) is the negative root of the following characteristic equation

\[
\frac{1}{2} \sigma^2 \beta (\beta - 1) + \eta \beta - \rho = 0,
\]

and \( d_2 = \frac{\ln g - \sigma^2(T-t) + (\frac{\eta}{\sigma^2} - \frac{1}{2})^2 + \frac{2\rho}{\sigma^2}}{\sigma \sqrt{T-t}} \), \( N(-d_2) = \left( \frac{1}{\sqrt{2\pi}} \right) \int_{-\infty}^{-d_2} e^{-\sigma^2/2} d\sigma \)

and \( 0 \leq N(d) \leq 1 \) is the cumulative normal distribution function. Note that as \( T \) approaches infinity, \( N(-d_2) = 1 \) and the option to discontinue research becomes a perpetual option case; however, as \( T \) approaches zero, \( N(-d_2) = 0 \) if \( g > 1 \). As the professor is near retirement, the value of the option to discontinue research approaches zero because he is going to retire soon.\(^8\)

We can now write an equation for the activity condition \( V^S - V^R = W \) where \( g \) is the productivity threshold at which the professor decides to discontinue research:

\(^7\) For readers who would like to study the rapidly developing literature of real options and optimal stopping applications in economics for the past two decades, see Dixit and Pindyck (1994) and Stokey (2008).

\(^8\) Note that the \( A_2 \) parameter is also a function of \( (1-e^{-(\rho-\eta)(T-t)}) \) and comes from the value-matching/smooth-pasting conditions. The option therefore also approaches zero as \( T \to t \). See equation (13) for details.
where $\rho > \eta$. The left-hand side of equation (10) shows the benefits of discontinuing research and the right-hand side the cost. The benefits consist of the expected discounted disutility of doing research, which the professor avoids by not doing research. The costs consist of the sum of the sacrificed expected discounted utility of the performance-related pay, the value of the real option to discontinue research in the future, and the one-off no-pecuniary penalty $W$ – or peer pressure – imposed by colleagues when a professor stops doing research.9

A professor may continue doing research even when the performance-related pay no longer compensates for the disutility from doing research. For someone who either dislikes doing research – $\gamma$ is positive and high – or is not very productive – $g$ is low – or is not paid very much for his research output – $w_b$ is low – or expects his research output to decline – $\eta$ is negative – it may nevertheless be optimal to continue doing research because of the possibility that productivity may improve in the future, the real option value is large ($\beta_2$ is negative so that $g^{\beta_2}$ is higher the lower is the value of $g$). However, as the professor nears retirement, both the benefits from continuing research and the real option approach zero, as does the discounted disutility of research effort.

Equation (10) has several intuitive implications. Clearly, when productivity $g$ is sufficiently high, the professor will continue doing research. The critical productivity level $g_2$ is higher

\begin{itemize}
  \item[a)] the lower is the performance-related pay $w_b$,
  \item[b)] the greater is the disutility of doing research $\gamma$,
  \item[c)] the weaker is peer pressure from colleagues $W$,
  \item[d)] the smaller is the rate of growth of research productivity $\eta$, and
  \item[e)] the lower is the level of uncertainty $\sigma$.
\end{itemize}

More importantly for our purposes, the critical productivity threshold depends on the age of the workers;

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9 See footnote 4 on the case of a constant peer pressure.
f) the older the worker, the higher is the productivity threshold at which he becomes inactive for $\eta > 0$, except for workers who are to retire very soon.

The intuition for the age effect is the following: Due to expected growth in research productivity $\eta$ the performance-related pay is expected to rise, as well as the real option, while the disutility from doing research stays constant over time. Hence, from the current perspective, the final year’s expected performance pay and possible research successes (the real option) count more in the professor’s current decision than the expected disutility of work. It follows that as the professor gets older the cost of giving up research declines more rapidly than the benefits and he is more likely to decide to become inactive in research for a given slump in research productivity. The young, in contrast, may decide to continue doing research because they have more time to enjoy the fruits of higher productivity and unexpected successes and this justifies current effort.

3.4 Numerical simulations

In order to further analyse the properties of the activity condition we run some numerical simulations. Before doing so, we need to determine the value of $A_2$ from the smooth-pasting condition,

$$w_b \left( 1 - e^{-(\rho - \eta)(T-t)} \right) = A_2 \left( \frac{\partial (g^\beta N(-d_2))}{\partial g} \right) = A_2 \left[ \beta g^\beta N(-d_2) - \frac{1}{\sqrt{2\pi}} \frac{g^\beta}{\sigma \sqrt{T-t}} e^{-d_2^2/2} \right].$$

The equation yields a solution for $A_2$, which is the following equation,

$$A_2 = \frac{w_b \left( 1 - e^{-(\rho - \eta)(T-t)} \right)}{\rho - \eta} \left[ -\beta g^\beta N(-d_2(g)) + \frac{1}{\sqrt{2\pi}} \frac{g^\beta}{\sigma \sqrt{T-t}} e^{-d_2^2/2} \right].$$

And the value of the real option to discontinue research is then as follows:

$$A_2 g^\beta N(-d_2(g)) = \frac{w_b g \left( 1 - e^{-(\rho - \eta)(T-t)} \right) N(-d_2(g))}{(\rho - \eta) \left[ -\beta N(-d_2(g)) + \frac{1}{\sqrt{2\pi}} \frac{e^{-d_2^2/2}}{\sigma \sqrt{T-t}} \right]}.$$
To further analyse the relationship between research effort and age we will perform numerical simulations on equations (10) and (13).

The figures below show the activity threshold derived from simulation results using benchmark values listed at the bottom of the figures. The left-hand panel of Figure 1 shows that the activity productivity threshold is increasing in age when $\eta > 0$ until the professor is just about to retire when it falls abruptly.\(^{10}\) This implies that as the professor gets older he needs a higher level of research productivity to justify continued research. A slump in research productivity – perhaps a sequence of rejections from academic journals – is hence more likely to convince the older workers to discontinue research and become inactive. The right-hand panel of Figure 1 shows how an increase in peer pressure $W$ lowers the threshold so that it takes a greater slump in research productivity to convince the professor to stop research. Figure 2 shows that increased uncertainty about future research output $\sigma$ has the same effect of shifting the thresholds downwards, as does an increase in the rate of performance-related pay $w_b$.

The fall in the threshold close to retirement is caused by the non-zero cost of discontinuing research $W$. A professor will not want to attract the scorn of his colleagues for stopping research for a very short period of time. This is apparent in the right-hand panel of Figure 1.

\(^{10}\) The effect is reversed when growth $\eta$ is negative, the younger professors may then decide to become inactive at higher levels of productivity than the older ones.
**Figure 1.** Productivity growth, peer pressure and the inactivity threshold

![Figure 1](image1.png)

**Figure 2.** Uncertainty, performance-related pay and the inactivity threshold

![Figure 2](image2.png)

Parameter values: $\sigma = 0.2$, $\rho = 0.1$, $\eta = 0.02$, $w_b = 1$, $W = 0.2$, $t = 0$, $\gamma = 1.0$, and age = 65 - $T$. Note that for a professor with ten years to retirement at the age of 55 the value of $T$ is equal to 10.

### 4. Monitoring

We now change the model in order to allow for monitoring of research effort. As in the efficiency wage literature, we assume that the departmental chair can observe research effort, in addition to research output, but only at a cost. Research effort can thus be checked regularly and the professor gets fired if caught shirking his research duties.

Following Shapiro and Stiglitz (1984) we assume a Poisson detection technology and let professors who do not do research face a constant probability $q$ of being fired. This makes the research inactive professor discount his future wages $w_0$ at rate $\rho + q$ when not doing
research, while the discount rate remains unchanged at \( \rho \) when doing research. This addition to the model changes equation (4) to

\[
(4') \quad V^s = E \left[ \int_{t}^{T} w_0 e^{-(\rho+q)(T-t)} \, ds \right] = \frac{w_0 \left(1 - e^{-(\rho+q)(T-t)}\right)}{\rho + q}
\]

and we get two new terms on the right-hand side of equation (10);

\[
(10') \quad \gamma \left(1 - e^{-\rho(T-t)}\right) = \frac{w_0 g \left(1 - e^{-(\rho-q)(T-t)}\right)}{\rho - \eta} + A_2 g^{\beta_2} N \left(-d_2 \left(g\right)\right) + K
\]

Those who shirk their duties discount future wages \( w_0 \) at a higher discount rate because they face the probability \( q \) per unit time of being fired by their departmental chair. Hence the difference between the last two terms on the right-hand side of the equation is positive and measures the value of job security, which is reduced when a professor decides to stop doing research. The cost of not doing research now becomes the sum of four terms: The sacrificed performance-related pay; the sacrificed option to stop research at a later time; the negative response of colleagues; and reduced job security. Figure 3 shows the activity thresholds that have become steeper with monitoring and remain upward sloping even for the case of \( \eta = 0 \). The slope of the threshold is steeper because the value of job security is falling in age – workers close to retirement have less to lose from being fired since they would have quit their job soon anyway.11

An older professor who has suffered setbacks in research – experienced a lower level of \( g \) – would hence be more likely to become inactive than a younger professor because he has less time left to recover his productivity and enjoy unexpected research results, as described in Section 2, and, moreover, he has less to fear from his employer since he is going to retire soon anyway.

\[\text{11 This leaves out any reputational effects that could have an offsetting effect and also any adverse effect on pension rights.}\]
It follows from the analysis that older research-active professors either enjoy doing research better than their younger colleagues or are more productive on average. It follows that while there are many inactive older professors the active professors tend to be quite productive. This is in accordance with the empirical results of Oster and Hamermesh (1998) who find that, comparing authors age 36-50 to those over 50, the degree of heterogeneity in terms of research productivity increases with age.

5. Tenure

Finally, we allow for a tenure effect by letting $q$ be declining in age. In this case workers face increased job security – a falling probability of detection and dismissal – the older they get. We assume the time profile for $q$ shown in Figure 4 and captured by the logistic function where job security increases initially at an increasing rate but then stabilizes around the age of 50. The probability of firing, $q$, is assumed to be highest at age 20 and equal to 0.25, and lowest when the worker approaches retirement and equal to 0.05. The reflection on the second derivative happens at age 40. The time profile is described by the following logistic function:
\[ q = 0.25 - \frac{0.2}{1 + \exp(-0.3 \times (\text{age} - 40))}. \]

According to this function, \( q \) is near 0.25 for young workers, and 0.05 for old workers who are close to retirement.

**Figure 4.** The effect of age on job security

![Figure 4](image)

Figure 5 then shows the activity thresholds for both \( \eta = 0.0 \) and \( \eta = 0.02 \). Note that the threshold is upward-sloping in both cases, more so when \( \eta = 0.02 \).

**Figure 5.** Activity thresholds with a tenure effect

![Figure 5](image)

Parameter values the same as in Figures 1 and 2.
Compared to the thresholds in Figure 3, the difference between the value of the new threshold for the young and the old workers is much greater in this case. The tenure effect weakens the incentive for the old workers to continue with research further. The old, unproductive professor may now decide to enjoy leisure on the job by discontinuing research because he has little time left to attain higher productivity; it does not matter much if he is found out and dismissed; and the chances that he be dismissed are low because of his tenured position.

6. Empirical predictions

The empirical prediction coming from our model is that the frequency of inactivity among older university teachers is higher than that for the younger ones because they value the real option to stay research active less. In contrast, the Ben-Porath model would predict that all university teachers slow down as they approach retirement – this is the human capital effect. Our option value effect predicts that the ones who are successful – or really enjoy their work – continue doing research until retirement in spite of the human capital effect.

In this section we use data on research output from the University of Iceland to test the two predictions. The payroll division of the university uses data on research activity to calculate a single measure of activity for each member of staff per year. Activities such as publishing papers in academic journals and books, seminar presentations, conference attendance and so forth each give a fixed number of points each which are then summed up to generate one grand total for each member of staff (see Appendix II). The same point system is used for all departments which enables us to study research activity for the whole university. We use data for the calendar year 2008 when 640 members of staff were assessed. Tables AII-1 and AII-2 in Appendix II summarise the data and Figure AII-1 show the distribution of output for eight age groups. We can see from the distributions in the appendix the large number of inactive university teachers in the older age groups.

In Table 1 we show the number of research inactive member of staff for each age group. We note that this is higher for the oldest three groups than for the younger groups and more than doubles as a proportion of the number of people in each group between the 60-64 and
65-70 years groups. In the 65-70 group we find that 30% are inactive. Since the point system measures not just the number of articles published in academic journals but also working papers, seminar presentations and so forth, these individuals can be said to be completely inactive when it comes to research.

Table 1. The number of inactive members of staff

<table>
<thead>
<tr>
<th>Age</th>
<th>30-34</th>
<th>35-39</th>
<th>40-44</th>
<th>45-49</th>
<th>50-54</th>
<th>55-59</th>
<th>60-64</th>
<th>65-70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inactive</td>
<td>1</td>
<td>5</td>
<td>9</td>
<td>6</td>
<td>14</td>
<td>23</td>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>Total</td>
<td>20</td>
<td>45</td>
<td>70</td>
<td>91</td>
<td>130</td>
<td>137</td>
<td>88</td>
<td>60</td>
</tr>
<tr>
<td>%</td>
<td>5</td>
<td>11.1</td>
<td>12.9</td>
<td>6.6</td>
<td>10.8</td>
<td>16.8</td>
<td>13.6</td>
<td>30</td>
</tr>
</tbody>
</table>

Of the 18 inactive individuals in the oldest group we find that 14 are men (31% of all men in the age group) and 4 women (26.7% of all women in the age group).

In Table 2 we present results from the estimation of an equation where output (measured in points in year 2008) is regressed on dummy variables for eight age groups. The objective is to map the age profile of research. The results suggest that research output when both sexes are combined is rising until the early forties and then declining. Men slow significantly down in their fifties and sixties, so much that the average number of points drops from around 36 to about 18 per years or by 50%. Women also peak in their early forties at around 25 points but only decline down to 18 in their late sixties. It follows that men tend to produce more in their thirties and forties but lose their edge in their fifties and sixties. However, the null hypothesis that research output remains the same throughout life can only be rejected for men at the 5% level of significance, not for women and not for all workers groups together.12

In the second column we include only the research active. For the whole sample research activity is now no longer declining in age – although peaking in the early 40s – and for women it is rising so that output in the 65-70 age group exceeds that in the 30-39 age

---

12 A Wald test for the equality of all coefficients yields F= 1.64 (probability =0.12) for all members of staff; F= 2.21 (probability of 0.03) for men; and F=0.61 (probability of 0.75) for women. Hence, only in the case of men is the equality rejected.
The equality of coefficients can no longer be rejected for men at the 5% level of significance although it can still be rejected at the 10% level. There is now no longer any significant fall in research output from the late fifties to late sixties for men. This suggests that it is to some extent the research inactive in later working life that pull down average research activity in these years. These results provide support for the option value effect. We should note that the results are robust to the exclusion of any one of the academic departments. Finally, the last column has a (censored) Tobit regression on the whole sample which gives similar results to the least squares estimation on the whole sample.

In Table 3 we redo the estimation of Table 2 but control for academic departments and academic position.

The Wald test for the equality of all coefficients yields $F= 1.04$ (probability =0.40) for all teachers; $F= 1.99$ (probability of 0.06) for men; and $F=0.73$ (probability of 0.64) for women. We can reject equality for men at the 10% level of significance but we cannot reject equality in the case of women nor the whole sample at all.

---

**Table 2. Age, sex and research output**

<table>
<thead>
<tr>
<th>age</th>
<th>All*</th>
<th>Active</th>
<th>Tobit**</th>
<th>All*</th>
<th>Active</th>
<th>Tobit*</th>
<th>All*</th>
<th>Active</th>
<th>Tobit***</th>
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<td>(3.48)</td>
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<td>(5.70)</td>
<td>(5.81)</td>
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<td>35.96</td>
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<td>(6.77)</td>
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<td>(7.36)</td>
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<td>0.04</td>
<td>0.04</td>
<td>-</td>
<td>0.01</td>
<td>0.01</td>
<td>-</td>
</tr>
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<td>640</td>
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<td>342</td>
<td>411</td>
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<td>210</td>
<td>229</td>
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*) White heteroskedasticity-consistent standard errors and covariance, t-statistics in parentheses. **) z-statistics in parentheses.
### Table 3. Research output, academic departments and academic positions

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<tr>
<th>Age Group</th>
<th>All (1)</th>
<th>All (2)</th>
<th>Active (3)</th>
<th>Tobit (4)**</th>
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</table>

<table>
<thead>
<tr>
<th>Academic Departments</th>
<th>Social and human sciences</th>
<th>Social work</th>
<th>Economics</th>
<th>Law</th>
<th>Political science</th>
<th>Business Administration</th>
<th>Nursing</th>
<th>Pharmaceutical sciences</th>
<th>Medicine</th>
<th>Food science and nutrition</th>
<th>Psychology</th>
<th>Odontology</th>
<th>Languages, literature and linguistics</th>
<th>Theology and religious studies</th>
<th>Icelandic and comparative cultural st.</th>
<th>History and philosophy</th>
<th>Sport, leisure st. and social education</th>
<th>Teacher education</th>
<th>Educational studies</th>
<th>Industrial-, mech. eng. and comp.s.</th>
<th>Earth sciences</th>
<th>Life and environmental sciences</th>
<th>Electrical and computer engineering</th>
<th>Physical sciences</th>
<th>Civil and environmental engineering</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8.73 (1.56)</td>
<td>-6.89 (0.93)</td>
<td>3.84 (0.42)</td>
<td>-2.89 (0.44)</td>
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<td>-11.66 (1.96)</td>
<td>-9.95 (1.19)</td>
<td>-11.60 (2.46)</td>
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<td>10.09 (1.49)</td>
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<td>2.21 (0.32)</td>
<td>-9.46 (1.66)</td>
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<td>-2.59 (0.49)</td>
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<td>-2.33 (0.28)</td>
<td>-8.34 (1.07)</td>
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<td>-11.33 (1.16)</td>
<td>-16.13 (2.13)</td>
<td>-18.18 (2.14)</td>
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<td></td>
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<td>1.86 (0.21)</td>
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<td>-16.42 (2.04)</td>
<td>-25.01 (2.59)</td>
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<td>-7.91 (0.58)</td>
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<td>-8.69 (1.01)</td>
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<tr>
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<td>-1.20 (0.14)</td>
<td>-24.04 (2.76)</td>
<td>-2.34 (0.29)</td>
<td>-11.78 (1.57)</td>
<td>-7.73 (0.65)</td>
<td>-22.71 (2.93)</td>
<td>-1.41 (0.16)</td>
<td></td>
</tr>
</tbody>
</table>

| Position             | Professor | 16.98 (3.26) | 17.88 (3.29) | 18.69 (3.34) |
|                      | Associate professor | 3.62 (0.72) | 4.43 (0.84) | 3.29 (0.60) |
|                      | Assistant professor  | -5.78 (1.13) | -2.49 (0.46) | -8.92 (1.58) |

| R-squared            | 0.11 | 0.20 | 0.17 | - |
| Observations         | 640 | 640 | 552 | 640 |

*) White heteroskedasticity-consistent standard errors and covariance, t-statistics in parentheses. **) z-statistics in parentheses.
We find the same age pattern of research as in Table 2. In addition, we find that business administration is weak, as well as several other departments such as nursing, pharmaceutical sciences, medicine, languages and theology.

In column (2), we also test for a tenure effect – professors may reduce research output because they have more job security and find that professors at the University of Iceland have almost 17 points more on average than other members of staff. The inactivity phenomenon is hence only associated with age, not rank. In column (3) we only include the research active and find, as before, that the slowdown in output in later years is now smaller, although statistically significant ($F=2.73$). As in Table 2 the fall in output between the fifties and the sixties is now almost eliminated while the fall from the early forties to late forties and fifties is still there.

We finally run a probit regression on the binary variable for inactivity (inactivity gives the value one to the variable, otherwise zero), which we explain with age and dummy variables for professors, associate professors and assistant professors. The results in Table 4 confirm that the older workers are at a greater risk of becoming inactive.

<table>
<thead>
<tr>
<th>Table 4. Inactivity and age</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
</tr>
<tr>
<td>----------------------------------</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Constant</td>
</tr>
<tr>
<td>Age</td>
</tr>
<tr>
<td>professor</td>
</tr>
<tr>
<td>assoc. professor</td>
</tr>
<tr>
<td>assist. professor</td>
</tr>
<tr>
<td>R-squared</td>
</tr>
<tr>
<td>Observations</td>
</tr>
</tbody>
</table>

Maximum-likelihood estimation: Logit.

From the coefficient of “age” we can calculate that each additional year raises the probability of being inactive by 0.17% for men and 0.28% for women. However, women are less likely to become inactive from the outset as revealed by the constant term. Somewhat surprisingly,
assistant professors are more likely to be inactive than professors.\textsuperscript{14} Our results suggest that it is age, not status or job security, which affects the probability that someone becomes research inactive.

We have found that older individuals are more likely to become research inactive than their younger colleagues. We have also found that the high level of inactivity among older workers accounts completely for the observed slowdown in average output of female workers and also for most of the slowdown in research output observed for men in their sixties. However, we should note that the explanatory power of the equations is limited (R-squared no higher than 0.20) which implies that other factors than age are important for the research productivity of each individual. This leads us to the main weakness of the cross-section estimation which is that some of the omitted variables may be correlated with age. In order to address this problem we perform panel estimation in the following section.

7. Accounting for individual heterogeneity

The regression analysis in the previous section suffers from the limitation that the age-productivity relationship may be caused by individual heterogeneity. In order to control for this the panel regression reported in Table 4 was performed using data on research output for 719 individuals in the seven years from 2003 to 2009.\textsuperscript{15}

The first column of Table 5 reports the results when all members of staff are included. The results are consistent with the cross-section results in that the coefficient of age is statistically significant and negative while the productivity of professors is greater than that of other members of staff. In the second column we add a dummy variable for men and also interact it with age. In line with our previous results we find that the dummy variable has a significant and positive coefficient while the interaction between age and sex implies that men slow down faster than women. Finally, in the third column we redo the estimation and this time include only research active individuals. Again, consistently with our previous

\textsuperscript{14} We should note that the age profile is similar for the three academic positions; the average age of professors in the sample was 56, the average age of associate professors 52 and the average age of assistant professors was 49.

\textsuperscript{15} We would have liked to go further back than 2003 but this is not possible due to data limitations.
results, we find that the rate at which male members of staff slow down is lower when the inactive individuals have been omitted. As before, professors tend to be more productive than associate and assistant professors.

Table 5. Panel regression

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</thead>
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</tr>
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<td>R-squared</td>
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</table>

Random effects estimator: Swamy and Arora estimator of component variances. * indicates significance at the 5% level. Departmental controls were included.

The explanatory power of the equation is rather limited. The weighted R-squared is only 0.06 and the unweighted one 0.21 for the whole sample and 0.05 and 0.17 for the research active sample. However, omitted variable should not bias our estimates in this case due to the panel setup.

8. Conclusions

Following a string of setbacks in research, it takes a higher level of productivity to convince an older professor to discontinue research than a younger one. The young can look forward to a long career that may generate rising productivity and unexpected successes while the older
ones are closer to retirement and have less to lose from discontinuing research. The older professor may also be less threatened by the prospect of being dismissed in the light of low research effort because he has little time left before retirement and because of his tenured position.

Looking at data on research output from the University of Iceland, we find that the risk of becoming inactive is rising in age and the higher proportion of inactive workers in the oldest cohorts explains their lower average research output. We find that a substantial part of the reason why groups of individuals appear to slow down in terms of research when they approach retirement is that some of them become entirely inactive while others continue as before. However, the increased job security of professors does not reduce research; professors tend to be considerably more productive than either associate professors or assistant professors.

Our results suggest that becoming inactive in research can be optimal for university teachers when faced with setbacks given the external environment and salary scheme of the university. For this reason universities can induce their older faculty to remain research active by striving to make their research more productive and enjoyable, maintaining peer pressure, reducing job security and offering higher performance related pay.

The intuition of the model is applicable to other professions where a part of a worker’s effort is not observable by the employer. These results can explain why firms can profit from incentive schemes aimed at promoting the productivity of older workers; in particular these workers require a higher real wage to deter them from shirking their duties. Our model thus complements the work of Lazear (1979) by showing how rising wage profiles can be used to increase a worker’s effort but for a different reason. In our model rising wages are needed to offset an increasing temptation to shirk one’s duties as the day of retirement nears while in Lazear’s paper rising wages make workers have higher effort levels throughout their careers in the hope that they will keep their jobs and live to higher wages in late career.
References


**Appendix I: Derivation of Equation (8)**

The corresponding integral for equation (7) in the text is denoted by

\[
(A1) \quad V^R = E \left[ \int_0^T \left( w_0 + w_b g_s - \gamma \right) e^{-\rho(t-s)} ds \right].
\]

Directly integrating (A1) without considering the possibility of shirking in research gives the following particular solution to \( V^R \)

\[
(A2) \quad V^R = \frac{(w_0 - \gamma) \left(1 - e^{-\rho(T-t)}\right)}{\rho} + \frac{w_b g_s \left(1 - e^{-(\rho-\eta)(T-t)}\right)}{\rho - \eta}.
\]

Substituting (A2) back into equation (7) in the text shows (A2) is the correct particular solutions. The homogenous part of equation (7) in the text has the following form:

\[
(A3) \quad \rho V^R = \eta g V^R_g + \frac{1}{2} \sigma^2 g^2 V^R_{gg} + V^R_t
\]

Chen and Zoega (2010) have shown the detailed derivations of the solution for an equation similar to (A3). We use another way to show how to obtain real options for discontinuing
research. It is commonly known that the perpetual real options have the functional form of component \( g^\eta \) and the corresponding characteristic equation without considering \( v_R \) is as follows,

\[(A4) \quad \frac{1}{2} \sigma^2 \beta (\beta - 1) + \eta \beta - \rho = 0.\]

As we only consider the opt-out (shirking) options, we only need to choose the negative root of equation (A4),

\[(A5) \quad \beta_2 = \frac{1}{2} - \frac{\eta}{\sigma^2} - \sqrt{\left( \frac{\eta}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2 \rho}{\sigma^2}} < 0.\]

Positive root for beta is not chosen in order for the real options not to approach infinity when productivity becomes very big. It is then natural to guess that real options to equation (A3) have the following functional form

\[(A6) \quad V^R(g,t;T) = A_2 g^{\beta_2} N(-d_2(g,t;T)),\]

where \( A_2 \) is the unknown parameter, and

\[(A7) \quad d_2 = \frac{\ln g - \sigma^2(T-t) \sqrt{\left( \frac{\eta}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2 \rho}{\sigma^2}}}{\sigma \sqrt{T-t}},\]

\[(A8) \quad N(-d_2) = \left( \frac{1}{\sqrt{2\pi}} \right) \int_{-\infty}^{-d_2} e^{-\sigma^2/2} d\sigma.\]

The \( d_2 \) function has the components of \( \beta_2 \) from perpetual real options to shirk. We can then prove that (A6) is one of possible solutions to (A3) by plugging (A6) back to (A3).

Differentiation \( V^R(g,t;T) = A_2 \left( \frac{1}{\sqrt{2\pi}} \right) g^{\beta_2} \int_{-\infty}^{-d_2} e^{-\sigma^2/2} d\sigma \) by using Leibnitz rule gives

\[(A9) \quad \eta g V^R_{\eta} = \eta \left[ \beta_2 g^{\beta_2} \int_{-\infty}^{-d_2} e^{-\sigma^2/2} d\sigma - g^{\beta_2} \frac{1}{\sigma \sqrt{T-t}} e^{-d_2^2/2} \right] \frac{A_2}{\sqrt{2\pi}},\]

\[(A10) \quad V^R_t = \left[ \frac{\ln g}{2\sigma(T-t) \sqrt{T-t}} - \frac{\sigma}{2 \sqrt{T-t} \sqrt{\left( \frac{\eta}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2 \rho}{\sigma^2}}} \right] \frac{A_2}{\sqrt{2\pi}} g^{\beta_2} e^{-d_2^2/2},\]
Substituting (A6), (A9) – (A11) back to equation (A3) and collecting terms gives

\[
\frac{1}{2} \sigma^2 g^2 V^R_{gg} = -\frac{\sigma}{2\sqrt{T-t}} \beta_2 \frac{A_1}{\sqrt{2\pi}} g^{\beta_1} e^{-d_z^2/2} + \frac{\sigma^2}{2} \beta_2 (\beta_2 - 1) \frac{A_2}{\sqrt{2\pi}} g^{\beta_2} \int_{-\infty}^{d_z} e^{-\sigma^2/2} d\sigma
\]

(A11) \[\begin{aligned}
&+ \frac{1}{2(T-t)} \frac{A_2}{\sqrt{2\pi}} g^{\beta_2} \left[ \ln g - \sigma^2 (T-t) \left( \frac{\eta}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2\rho}{\sigma^2} \right] \frac{\sigma}{\sqrt{1+T-t}} e^{-d_z^2/2} \\
&- \frac{\sigma}{2\sqrt{T-t}} (\beta_2 - 1) \frac{A_2}{\sqrt{2\pi}} Y^{\beta_2} e^{-d_z^2/2}.
\end{aligned}\]

Substituting (A6), (A9) – (A11) back to equation (A3) and collecting terms gives

\[
\frac{\sigma^2}{2} \beta_2 (\beta_2 - 1) + \eta \beta_2 - \rho \frac{A_1}{\sqrt{2\pi}} Y^{\beta_2} \int_{-\infty}^{d_z} e^{-\sigma^2/2} d\sigma
\]

(A12) \[\begin{aligned}
&+ \left[ -\beta_2 + \frac{1}{2} - \frac{\eta}{\sigma^2} - \left( \frac{\eta}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2\rho}{\sigma^2} \right] \frac{\sigma}{\sqrt{T-t}} \frac{A_2}{\sqrt{2\pi}} Y^{\beta_2} e^{-d_z^2/2} = 0
\end{aligned}\]

The items in two brackets are equal to zero due to equation (9) in the text (or (A4)) and (A5), which concludes the proof that \( V^R(g,t;T) = A_2 g^{\beta_2} N(-d_z(g,t;T)) \) is the general solutions to equation (7) in the text. Combining the particular solutions and homogenous solutions [A(2) and (A6)] gives equation (8) in the text.
Appendix II. University of Iceland point system for research (Since 2003)

A1. Dissertations
A1.1 Candidate- or masters thesis (15 points)
A1.2 Doctoral thesis (30 points)

A2. Books
A2.1 Books, academic (10-60 points)
A2.2 Books, republications (0-10 points)

A3. Academic articles in journals
A3.1 Article in internationally acknowledged journals cited in ISI Web of Science (15 points)
A3.2 Article in other refereed journals (10 points)
A3.3 Other material in refereed journals (0-5 points)
A3.4 Article in a non-refereed journal (0-5 points)

A4. Papers in refereed conference proceedings and book chapters
A4.1 Paper in a refereed conference proceedings (5-10 points)
A4.2 Book chapter (5-10 points)

A5. Other academic activity
A5.1 Scientific report or memorandum (0-5 points)
A5.2 Book review (1-2 points)
A5.3 Lectures
   5.3.1 Lecture at scientific conference (3 points)
   5.3.2 Lecture for the academic community (1 point)
   5.3.3 Plenary lecture or keynote address at an international conference (5 points)
A5.4 Posters
   5.4.1 Poster at a scientific conference (2 points)
   5.4.2 Poster at other meetings (1 point)
A5.5 Translations (0-10 points)
A5.6 Other (software, patents, psychological tests, bills, design projects etc.) (0-10 points)

A6. Citations in ISI Web of Science
First 10 citations: 1 point/citation
Next 20 citations: 0,5 point/citation
Citations exceeding 30: 0,1 point/citation.

7. Editorial work, academic publications
7.1. Editor of an academic journal (2-5 points/year)
7.2. Member of editorial board of an academic journal (1-2 points/year)
7.3. Editor of an academic book (2-5 points)
7.4. Member of editorial board of an academic book (1-2 points)

In the case of multiple author articles or books, the points are calculated using the following formula:
2 authors: 1,5 x points / 2, 3 authors: 1,8 x points / 3, 4 authors or more 2,0 x points / number of authors
### Table AII-1. Summary of data – Departments

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<tr>
<th>Schools</th>
<th>Departments</th>
<th>Staff members</th>
<th>Men</th>
<th>Women</th>
<th>Average age</th>
<th>Research output - average points -</th>
<th>Research output - standard deviation -</th>
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Table AII-2. Summary of data – Institutes

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<th>Average age</th>
<th>Research output - average points -</th>
<th>Research output - standard deviation -</th>
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Figure AII-1. The distribution of output for different age groups

The left-hand panel shows the distribution of research output when everyone is included while the right-hand side panel show the distribution when the research inactive have been removed from the sample.