Horizontal Agreements and R&D Complementarities: Merger versus RJV

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Abstract

We study the decision of two firms within an oligopoly concerning whether to enter into a horizontal agreement to exploit complementarities between their R&D activities and, if so, whether to merge or form a research joint venture (RJV). In contrast to horizontal merger, there is a probability that an RJV contract will fail to enforce R&D sharing. We find that a horizontal agreement always arises. The insiders’ merger/RJV choice involves a trade-off: While merger offers certainty that R&D complementarities will be exploited, it leads to a profit-reducing reaction by outsiders on the product market, where competition is Cournot. Greater brand similarity and contract enforceability (“quality”) both favour RJV, while greater R&D complementarity favours merger. Interestingly, the insiders may choose to merge even when RJV contracts are always enforceable, and they may opt to form an RJV even when the likelihood of enforceability is negligible.

Keywords: horizontal merger, research joint venture (RJV), contract enforceability, process R&D, R&D complementarity.

JEL numbers: O30, L13, D43

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1 Introduction

Within an industry, there are often complementarities between rival firms’ R&D activities, in the sense that their research efforts are not perfect substitutes.\(^1\) It is thus natural that firms should seek to find methods of exploiting such R&D complementarities to their mutual benefit. In this paper, we examine the choice between two such methods, horizontal merger and research joint venture (RJV).\(^2\) Both horizontal merger and RJV facilitate the sharing of R&D results; however, in an RJV, the participating firms remain independent entities who maximise their own profits, whereas decisions within a merged firm are taken to maximise joint profits.

We model an oligopolistic industry, where firms compete both in process R&D and, subsequently, in outputs, and we focus on the decision of two firms concerning whether to enter into a horizontal agreement to exploit R&D complementarities and, if so, whether to merge or form an RJV.\(^3\) Surprisingly, to the best of our knowledge, this “merger versus RJV” decision has received no attention in the vast theoretical literature on the organisation of R&D. This paper fills that gap.

An episode in the corporate history of BP (formerly British Petroleum), the third-largest energy company in the world, nicely illustrates some of the salient issues in the merger/RJV decision. In 2000, the Federal Trade Commission approved the mega-merger between BP (then called BP Amoco) and the Atlantic Richfield Company (ARCO) partly on the grounds that, by committing the firms to sharing their accumulated technical expertise, it would significantly reduce extraction costs at the enormous Prudhoe Bay oil field in Alaska.

\(^1\)For example, Kamien et al. (1992, p.1298) characterise the R&D process as involving trial and error (“it is a multidimensional heuristic rather than a one-dimensional algorithmic process”), with each firm simultaneously pursuing several avenues of research, only some of which pay off.

\(^2\)Gugler and Siebert (2007), for example, find that both mergers and RJVs in the semiconductor industry are associated with substantial efficiency gains, which they partly attribute to the internalisation of positive R&D externalities. Andrade et al. (2001) provide empirical evidence that “synergies” from combining firms’ R&D stocks often motivate horizontal merger. For RJVs, Hernán et al. (2003) find that the potential for technologies to spill over to other firms positively influences RJV formation, and Veugelers (1998, p. 420) states that empirical evidence “suggests that exploiting complementarities is a major motive for cooperation in R&D”.

\(^3\)We focus on a single, bilateral agreement for simplicity.
Competition policy practitioners typically argue that an RJV is socially preferred to a horizontal merger because, while both institutional structures facilitate the exploitation of R&D complementarities, an RJV has the added virtue that it preserves product-market competition between the participants. In response to this objection, BP and ARCO persuasively argued that, in their case, over twenty years of contractual experiments to gain the benefits of information pooling had failed.\(^4\)

Our model aims to capture some of the features of the BP/ARCO case study. In particular, we develop the notion that, relative to an RJV, a horizontal merger is more “effective” at inducing participating firms to pool their R&D results. This was the BP/ARCO claim, and it is intuitively appealing because, within an RJV, firms, who remain independent, have a strong incentive not to reveal all of their R&D results to their partners. RJVs attempt to address this problem through the use of contracts that are signed when they are established, but such contracts might fail to be enforceable ex post (i.e. after R&D results have been realised). Therefore, for the insiders, a horizontal merger, which precommits them to pooling R&D outputs (through the objective of joint profit maximisation), can be an attractive alternative to forming an RJV – independently of market power considerations.

An innovation of our analysis with respect to the existing literature on RJVs is that we allow for the possibility that the RJV contract might fail to enforce R&D sharing by the insiders.\(^5\) This is consistent with the empirical evidence surveyed by Veugelers (1998, p. 420), which “suggests that cooperation carries a disturbingly high risk of failure”. Underlying this, R&D “verifiability” – that is, observability by courts – is a crucial element in our analysis. In advance of results emerging from the research lab, both the quantitative and the qualitative nature of R&D output are typically unknown. This makes it very tough to write an RJV contract that describes R&D results across all possible states of the world sufficiently precisely to

\(^4\)In the jargon, the efficiencies were “merger-specific”. For more information on the BP/ARCO merger, see Farrell and Shapiro (2001, p. 705), Bulow and Shapiro (2004), and http://news.bbc.co.uk/1/hi/business/712962.stm.

\(^5\)Another innovation is that we allow for the presence of outsider firms – i.e. the RJV covers only a subset of firms within the industry. In Section 5 we discuss the implications of this for welfare analysis.
ensure that courts can always enforce its sharing rules.\textsuperscript{6,7} The incomplete nature of RJV contracts means that there is always a chance that any given contract, which commits the insiders to sharing certain defined R&D outputs, will fail to be enforceable because the realised R&D results fall outside the terms of the original contract. We capture the success/failure of the RJV contract by assuming that it is enforceable with a certain probability, which we term the “contract quality”, and we assume that firms maximise expected profits.

Much of the remaining structure of our model is standard to facilitate straightforward comparisons with existing results in the literature. We consider an oligopoly containing initially-identical firms.\textsuperscript{8} The firms produce substitute brands of a differentiated good (the demand system is linear), and they compete à la Cournot on the product market. Each firm has a constant unit cost, which decreases at a constant rate in its own R&D investment. Within a horizontal agreement (either a merged firm or an RJV whose contract is enforceable), a firm can also access the other insider’s R&D, and our model contains a parameter that measures the degree of R&D complementarity (i.e. the extent to which a partner’s R&D decreases a firm’s own unit cost). Aside from such R&D sharing, there are no inter-firm R&D spillovers, however.

Existing applied theory papers on horizontal mergers and RJVs have tended to analyse the choice between a given type of horizontal agreement,

\textsuperscript{6}To illustrate why precision is necessary in the RJV contract, consider for example the case where the RJV relates to a single product, \( X \), but one of the RJV members is a multiproduct firm that operates a single R&D lab for all of its lines of business. Clearly, the multiproduct firm would challenge an RJV contract that simply stated “share all R&D related to \( X \)” by claiming that any given piece of its R&D output related to another product and so need not be disclosed within the RJV. Breadth is necessary in the RJV contract because, even if it perfectly describes previous R&D output, the participating firms have incentives to target potentially non-verifiable R&D activities.

\textsuperscript{7}Aside from the problem of contract enforceability, our modelling of behaviour within RJVs is identical to the “RJV competition” case of Kamien \textit{et al.} (1992), where RJV insiders set both their R&D and their output levels independently to maximise their own profits. In the Conclusion, we discuss how our modelling of RJVs relates to Kamien \textit{et al.}’s other case of “RJV cartelization” (i.e. co-operation between RJV insiders in the setting of R&D but competition in outputs), which could be interpreted as an attempt to solve the contractual problems we consider by contracting all R&D activities out to a third party.

\textsuperscript{8}Röller \textit{et al.} (2007) present evidence from the US that RJVs are most likely to form between firms of similar sizes.
either merger or RJV, and no-agreement. Thus, our paper, which focusses on the merger/RJV choice, is a bridge between those two literatures. Our paper also fills a gap between the IO literature on RJVs (e.g. Kamien et al., 1992), which typically assumes that RJV contracts are always enforceable, and that on incomplete contracts (e.g. Hart and Moore, 1990), which typically abstracts from the effects of product-market competition by assuming bilateral monopoly-type interactions between firms in unrelated (or vertically related) markets.

Our first finding in Section 4 is that, for the two insiders, an RJV always dominates the no-agreement outcome. This finding suggests that a horizontal agreement, either merger or RJV, will always arise in equilibrium. With identical firms, it is well established within the literature that industry profits are maximised when all firms determine their R&D investments co-operatively, and pool R&D results, but compete la Cournot in outputs; this set-up is termed RJV cartelization by Kamien et al. (1992). In contrast, our result relates to an RJV where R&D investments are determined non-cooperatively and participation is limited to a subset of firms; thus, our first result is a substantive contribution in itself.

Next, we investigiate the insiders’ merger/RJV choice. Our analysis here can be related to that of Davidson and Ferrett (2007), who examine the insiders’ choice between horizontal merger and no-agreement within the same modelling set-up as ours. Merger allows the exploitation of R&D complementarities, but the reaction that it provokes from outsiders on the product market (where competition is Cournot), who expand their production in response to the insiders’ attempts to restrict output and reach the monopoly solution, works to undermine its profitability. Therefore, a profitability
trade-off between merger and no-agreement emerges: greater R&D complementarity favours merger but greater brand similarity, which means stronger product-market interactions, favours no-agreement.

In our model, RJV resembles no-agreement in that, in both scenarios, all firms in the industry make both their R&D and their production decisions independently to maximise their own (expected) profits. However, unlike no-agreement, RJV does allow some probability that R&D complementarities will be successfully exploited. This is what lies behind our finding, discussed above, that the insiders always prefer RJV to no-agreement. It thus follows that the size of the region in parameter space where merger dominates no-agreement will be larger than that where merger dominates RJV.

The insiders’ merger/RJV choice entails a trade-off: while merger offers a higher probability than RJV that R&D complementarities will be successfully exploited, it tends to be less profitable, ceteris paribus, than RJV on the product market. In product-market competition, merger suffers in comparison with RJV for the same reason that it underperforms no-agreement: the profitability of merger is undermined by the reactions of outsiders. Therefore, to maintain indifference between merger and RJV on the part of the insiders, greater brand similarity (which heightens the exposure of the merged firm to the reactions of outsiders) must be counterbalanced by a lower contract quality (or probability of contract enforceability). We also show that a rise in the degree of R&D complementarity makes merger “more likely” relative to RJV, in the sense that the region in parameter space where merger is chosen grows in size.

Finally, we derive two surprising and interesting results that relate to the extremes of brand similarity. First, we show that the insiders may optimally choose to merge even when RJV contracts are “perfect” (i.e. always enforceable); this requires a low level of brand similarity. Secondly, on the contrary, we show that the insiders may optimally choose to form an RJV even when the probability of contract enforceability is negligible; this requires a high level of brand similarity.

The remainder of the paper is organised as follows. Section 2 presents

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(1983). In the Conclusion, we consider how our model would work with Bertrand (price) competition in the product market.

\(^{13}\)i.e. for a given vector of marginal costs across all firms.
our model, and Section 3 presents the equilibrium solutions for the three possible agreement choices by the insiders: no-agreement, horizontal merger, and RJV. Section 4 analyses the insiders’ no-agreement/merge/RJV choice, and Section 5 presents a brief illustrative welfare analysis. Finally, Section 6 concludes by relating our results back to the BP/ARCO case study and considering some possible extensions.

2 Model

There are $N$ (initially identical) firms, each producing a single brand of a differentiated good. Firm $i$’s price and quantity are denoted by $p_i$ and $q_i$ respectively, and the brand inverse demand function is

$$p_i = a - q_i - \beta q_{-i}$$  \hspace{1cm} (1)

where $a$ is a constant, $\beta \in [0,1]$ measures brand similarity,\footnote{Thus, $\beta = 0$ represents independent goods, and $\beta = 1$ represents identical brands.} and $q_{-i} = \sum_{j \neq i} q_j$. ($\beta$ will also measure the strength of strategic interaction on the product market.) Initially, each firm operates with the same constant unit cost, $c$, and there are no fixed costs. There is no entry into the industry, and $a > c$.

Firms 1 and 2 have two possible means of exploiting complementarities between their R&D activities: horizontal merger (HM) and research joint venture (RJV). We consider the following sequence of moves: in stage 0, firms 1 and 2 choose between NA (no agreement), HM, and RJV; in stage 1, all firms simultaneously choose how much to invest in process R&D; and in stage 2, all firms compete à la Cournot on the product market. We assume complete information, and we solve the game backwards to isolate its subgame perfect Nash equilibrium.

The stage 0 choice between NA, HM and RJV affects the insiders’ (firms 1 and 2) costs and their subsequent behaviour. The NA case, where firms 1 and 2 remain independent, is the easiest to consider because all the firms in the industry are symmetric. Firm $i$’s total cost of R&D is $(\gamma/2)x_i^2$, where
\( \gamma \) is a constant. Under NA, this R&D expenditure lowers its unit cost to \( c - x_i \).\(^{15}\)

Under HM, the subsequent R&D and output decisions of firms 1 and 2 are made to maximise their joint profits. Horizontal merger guarantees that R&D complementarities will be exploited, so firm 1’s unit cost is \( c_1 = c - x_1 - \theta x_2 \) (an analogous expression holds for firm 2), where \( \theta \in [0, 1] \) measures the degree of R&D complementarity. (This specification for the insiders’ unit costs follows d’Aspremont and Jacquemin, 1988.) The outsiders under HM, firms 3, 4, ..., N, behave like the firms in the NA case.

Under RJV, firms 1 and 2 agree in a contract, signed before R&D investments are decided, to share their R&D results before the start of stage 2 (product market competition). However, after R&D results have been realised, this contract turns out to be enforceable only with probability \( \omega \), which measures “contract quality”. Thus, \( \omega \) can also be interpreted as the probability that R&D outputs are observable by courts, or “verifiable”.\(^{16}\) (In a nutshell, the problem is that the RJV contract, written in advance of R&D activity being undertaken, cannot exhaustively and precisely catalogue R&D output in every possible state of the world. Such breadth of coverage is necessary, however, because RJV insiders have a strong incentive not to share their research outputs.\(^{17}\)) We will sometimes refer to contract enforceability as “RJV success” and to contract unenforceability as “RJV failure”, and we assume that RJV success/failure becomes common knowledge.

If, in the light of the insiders’ R&D outputs, the RJV contract turns out to be unenforceable (with probability \( 1 - \omega \)), then the RJV insiders have a dominant strategy not to share their R&D results (because they can do so with impunity) and their unit cost functions in stage 2 are as under NA, e.g. \( c_1 = c - x_1 \). Alternatively, if the RJV contract turns out to be enforceable, then it will ensure that the insiders pool their R&D results and the insiders’ unit cost functions in stage 2 will be as under HM, e.g. \( c_1 = c - x_1 - \theta x_2 \).

\(^{15}\)There are no inter-firm spillovers from R&D. Essentially, we are concerned with cases where intra-firm R&D spillovers (within a horizontal agreement) are significantly greater than inter-firm R&D spillovers.

\(^{16}\)Note that we are assuming, for simplicity, that the R&D outputs of both insiders always have the same verifiability/non-verifiability status. Thus, it is impossible, for example, for firm 1’s R&D output to be verifiable (and its contractual commitment to share R&D therefore to be enforceable) while firm 2’s is not.

\(^{17}\)See, for example, the case discussed in the Introduction where an RJV contains a multiproduct firm.
In the RJV scenario, all firms (both insiders and outsiders) make both their R&D and their output choices independently to maximize their own expected profits (outsiders, of course, have access only to their own R&D results), and all firms are risk neutral.

3 Equilibrium solutions

3.1 No agreement between firms (NA)

In stage 2, the common first-order condition (FOC) for output is

\[ p_i - c_i = q_i \]  

(2)

where \( c_i = c - x_i \) and \( x_i \) is fixed. Solving the system of simultaneous equations, we get firm \( i \)'s equilibrium output:

\[ q_i = \frac{(2 - \beta) a + \beta C^F - \Delta_0 c_i}{(2 - \beta) \Delta_0} \]  

(3)

where \( C^F \equiv Nc - \sum_{i=1}^{N} x_i \) is the sum of marginal costs when firms 1 and 2 fail to share R&D, and \( \Delta_0 \equiv 2 + \beta (N - 1) \).

In stage 1, \( x_i \) is set to maximise \( \Pi_i = q_i^2 - \gamma x_i^2 \). Imposing symmetry \( (x_i = x_{NA} \forall i) \) on the resulting FOC and solving gives

\[ x_{NA} = \frac{2 (\Delta_0 - \beta) (a - c)}{(2 - \beta) \gamma \Delta_0^2 - 2 (\Delta_0 - \beta)} \]  

(4)

18In terms of the Kamien et al. (1992) taxonomy, this corresponds to the case of “RJV competition”. Having the RJV insiders compete both in R&D investments and in outputs also nicely captures the institutional fact that, in an RJV, the insiders remain independent firms (unlike following a horizontal merger).

19We report the equilibrium solutions for the NA and HM cases relatively briefly as they are covered in-depth in Davidson and Ferrett (2007).

20(3) can be derived by summing (2) over all firms to derive the equilibrium industry output and then substituting this back into (2).
as the common equilibrium level of R&D output. The common equilibrium level of profits, \( \Pi_{NA} \), can be found by substituting (4) into (3) and using \( \Pi_i = q_i^2 - \frac{\gamma}{2} x_i^2 \).

### 3.2 Horizontal merger (HM)

In stage 2, the insiders to the merger (firms 1 and 2) set \( q_1 \) and \( q_2 \) to maximise \( (p_1 - c_1) q_1 + (p_2 - c_2) q_2 \), while the outsiders behave like the firms in the NA game. Therefore, the FOCs for output are

\[
\begin{align*}
\begin{cases}
p_1 - c_1 = q_1 + \beta q_2 \\
p_2 - c_2 = q_2 + \beta q_1 \\
p_j - c_j = q_j & \forall j \geq 3
\end{cases}
\end{align*}
\]

(5)

where \( c_1 = c - x_1 - \theta x_2 \), \( c_2 = c - x_2 - \theta x_1 \), and \( c_j = c - x_j \forall j \geq 3 \); and all the \( x \)'s are fixed.

The equilibrium outputs are

\[
\begin{align*}
q_1 + q_2 &= \frac{(2-\beta)a + \beta C^S - \frac{\Delta_0}{\Delta_1} (c_1 + c_2)}{\Delta_0} \\
q_j &= \frac{(2-\beta)a + \beta C^S - \frac{\Delta_0}{\Delta_1} (c_1 + c_2) - \Delta_1 c_j}{(2-\beta) \Delta_0}, \forall j \geq 3
\end{align*}
\]

(6)

where \( C^S \equiv Nc - (1 + \theta) (x_1 + x_2) - \sum_{x \in S} x_j \) is the sum of marginal costs when firms 1 and 2 share R&D, and \( \Delta_1 \equiv \Delta_0 - \beta^2 \).

In stage 1, the merged firm sets \( x_1 \) and \( x_2 \) to maximise \( \Pi_1 + \Pi_2 = q_1^2 + q_2^2 + 2\beta q_1 q_2 - \frac{\gamma}{2} (x_1^2 + x_2^2) \), while the outsiders behave like the firms in

\footnote{There are two restrictions on \( \gamma \). The second-order condition is \( \gamma > \frac{2(\Delta_0 - \beta)^2}{((2-\beta)\Delta_0)^2} \), and the condition for positive equilibrium marginal costs \( (c > x) \) is \( \gamma > 2(\Delta_0 - \beta) a / (2-\beta) \Delta_0 \). We ensure that both conditions hold in what follows, along with analogous conditions for the HM and RJV cases.}

\footnote{It is clear from the FOCs in (5) that merger leads to a restriction of output by the insiders relative to the outsiders. For example, if \( p_1 - c_1 = q_1 \) (as for an outsider) then \( p_1 - c_1 < q_1 + \beta q_2 \), so \( q_1 \) will be reduced.}
the NA game. Differentiating then imposing symmetry \(x_1 = x_2 = x_{IN}^{HM}\) and \(x_j = x_{OUT}^{HM} \forall j \geq 3\) on the FOCs and solving gives\(^{23}\)

\[
\begin{align*}
  x_{IN}^{HM} &= \frac{m_2 m_4 - m_1 m_6}{m_2 m_5 - m_3 m_6} (a - c) \\
  x_{OUT}^{HM} &= \frac{m_1 m_5 - m_3 m_4}{m_2 m_5 - m_3 m_6} (a - c)
\end{align*}
\]

where

\[
\begin{align*}
  m_1 &= (1 + \beta) (2 - \beta) (1 + \theta) (\Delta_0 - 2\beta) \\
  m_2 &= \beta (1 + \beta) (1 + \theta) (N - 2) (\Delta_0 - 2\beta) \\
  m_3 &= 2\gamma \Delta_1^2 - (1 + \beta) (1 + \theta)^2 (\Delta_0 - 2\beta)^2 \\
  m_4 &= 2 (\Delta_1 - \beta) \\
  m_5 &= m_4 \beta (1 + \theta) \\
  m_6 &= \gamma (2 - \beta) \Delta_1^2 - m_4 (1 + \beta)
\end{align*}
\]

Equilibrium profits, \(\Pi_{IN}^{HM}\) and \(\Pi_{OUT}^{HM}\), are found by substituting from (7) into (6) and using \(\Pi_{IN}^{HM} = (1 + \beta) \left( q_{IN}^{HM} \right)^2 - \frac{\gamma}{2} \left( x_{IN}^{HM} \right)^2\) while \(\Pi_{OUT}^{HM} = \left( q_{OUT}^{HM} \right)^2 - \frac{\gamma}{2} \left( x_{OUT}^{HM} \right)^2\).\(^{24}\)

### 3.3 Research Joint Venture (RJV)

The RJV comprises firms 1 and 2. Unit costs depend on R&D investments in stage 1 and also on whether the RJV succeeds or fails. Therefore, the sum of marginal costs over all firms is \(C^S\) with probability \(\omega\) (the RJV succeeds) and \(C^F\) with probability \(1 - \omega\) (the RJV fails).

Stage 2 under RJV is qualitatively identical to stage 2 under NA; see (3) above. In stage 1 under RJV, firm 1 chooses \(x_1\) to maximize expected profits:

\(^{23}\)The FOC for \(x_1\) with symmetry imposed is \(2 (1 + \beta) q_{IN}^{HM} \cdot \partial (q_1 + q_2) / \partial x_1 = \gamma x_{IN}^{HM}\). To derive \(q_{IN}^{HM}\), the equilibrium output of an insider, impose symmetry on \(q_1 + q_2\) in (6) and divide by 2.

\(^{24}\)Here, \(q_{IN}^{HM}\) and \(q_{OUT}^{HM}\) are, as the notation implies, the equilibrium production levels of an insider and an outsider respectively.
\[ \omega \left( \frac{(2 - \beta) a + \beta C^S - \Delta_0 (c - x_1 - \theta x_2)}{(2 - \beta) \Delta_0} \right)^2 \\
+ (1 - \omega) \left( \frac{(2 - \beta) a + \beta C^F - \Delta_0 (c - x_1)}{(2 - \beta) \Delta_0} \right)^2 - \gamma \frac{x_1^2}{2} \tag{8} \]

with an analogous expression applying to firm 2; while an RJV outsider \((j \geq 3)\) chooses \(x_j\) to maximize

\[ \omega \left( \frac{(2 - \beta) a + \beta C^S - \Delta_0 (c - x_j)}{(2 - \beta) \Delta_0} \right)^2 \\
+ (1 - \omega) \left( \frac{(2 - \beta) a + \beta C^F - \Delta_0 (c - x_j)}{(2 - \beta) \Delta_0} \right)^2 - \gamma \frac{x_j^2}{2}. \tag{9} \]

Imposing symmetry \((x_1 = x_2 = x_{IN}^{RJV}\) and \(x_j = x_{OUT}^{RJV}\) \(\forall j \geq 3)\) on the resulting FOCs and solving gives

\[
\begin{align*}
x_{IN}^{RJV} &= \frac{r_2 r_4 - r_1 r_6}{r_2 r_5 - r_3 r_6} (a - c) \\
x_{OUT}^{RJV} &= \frac{r_1 r_5 - r_3 r_4}{r_2 r_5 - r_3 r_6} (a - c) \\
x_{RJV} &= \left\{ \begin{array}{c} x_{IN}^{RJV} \\ x_{OUT}^{RJV} \end{array} \right\} \tag{10} \end{align*}
\]

where

\[
\begin{align*}
r_1 &= (2 - \beta) [\Delta_0 - (1 + \theta \omega) \beta] \\
r_2 &= \beta (N - 2) [\Delta_0 - (1 + \theta \omega) \beta] \\
r_3 &= \frac{\gamma}{2} (2 - \beta)^2 \Delta_0^2 - (\Delta_0 - 2\beta) [(\Delta_0 - \beta) (1 + \theta \omega) - \theta (1 + \theta) \omega \beta] \\
r_4 &= (2 - \beta) (\Delta_0 - \beta) \\
r_5 &= 2\beta (1 + \theta \omega) (\Delta_0 - \beta) \\
r_6 &= \frac{\gamma}{2} (2 - \beta)^2 \Delta_0^2 - (2 + \beta) (\Delta_0 - \beta)
\end{align*}
\]

Equilibrium profits, \(\Pi_{IN}^{RJV}\) and \(\Pi_{OUT}^{RJV}\), are found by substituting from (10) into (8) and (9).
4 Results

4.1 RJV dominates NA

Our first finding is that, for the insiders, RJV always dominates NA.\textsuperscript{25} That is, $\Pi_{IN}^{RJV} \geq \Pi_{NA}$ for all parameter values, with strict inequality if both $\theta$ and $\omega$ are non-zero. This finding is illustrated in Figure 1, which compares R&D investments and profits under NA and RJV for specific parameter values. It is also evident from the numerical analysis presented in the Appendix, which is the subject of Section 5 below, and from extensive numerical simulations that the authors have undertaken.\textsuperscript{26} However, as is often the case in models of multi-stage competition such as the present one, the complexity of the equilibrium solutions means that an analytical proof is, unfortunately, beyond reach.\textsuperscript{27}

\textsuperscript{25}An analogous finding holds for society as a whole, as we show in Section 5.
\textsuperscript{26}Details available on request.
\textsuperscript{27}Moreover, the asymmetry of the RJV equilibrium in our model further complicates the NA/RJV comparison.
Despite this, the intuition for $\Pi_{\text{IN}}^{\text{RJV}} \geq \Pi_{\text{NA}}$ in Figure 1 is both clear and strong. First, note that if either $\theta$ or $\omega$ equals zero (i.e. there are no R&D complementarities or RJV contracts are guaranteed to fail), then the RJV game is identical to the NA game. This explains the common vertical intercepts in Figure 1.

Now assume that both $\theta$ and $\omega$ are strictly positive. We can interpret the message of Figure 1 ($\Pi_{\text{IN}}^{\text{RJV}} > \Pi_{\text{NA}} > \Pi_{\text{OUT}}^{\text{RJV}}$ and $x_{\text{IN}}^{\text{RJV}} > x_{\text{NA}} > x_{\text{OUT}}^{\text{RJV}}$) as follows. Assume, first, that every firm’s R&D level is fixed at $x_{\text{NA}}$ and that only outputs can be varied. If, in this situation, two firms enter into a contract to pool their R&D stocks (as in an RJV), then their marginal costs will fall with probability $\omega$ and their expected profits will rise as, if the RJV succeeds, they expand on the product market at the expense of the $N-2$ outsiders.

Next, allow R&D levels to vary. The rise in insider output just described increases the “marginal benefit” (or marginal variable profit) to an insider of R&D since cost reductions are now spread over a larger quantity of pro-
duction. Hence, both insider R&D investment and insider profits rise. The response of outsiders to higher R&D investment by the firms in the RJV is to retrench: they are squeezed on the product market, and cut their R&D levels. In turn, this reaction by the outsiders benefits the insiders.

It is clear that *ceteris paribus* (specifically, holding the R&D levels of outsiders fixed) the insiders would benefit from the move from NA to RJV. Our argument has sought to show that the reaction of outsiders – specifically, the fact that they cut R&D investment in response to being squeezed on the product market following the formation of an RJV – is also beneficial to the insiders. Thus, we have charted a path from the NA to the RJV equilibrium in Figure 1 along which an insider’s profits are always increasing.

In Figure 1, while $\Pi_{NA}$ is independent of $\omega$, $\Pi_{IN}^{RJV}$ is increasing in $\omega$. This feature holds generally throughout all of our numerical simulations, and it is the basis for our conclusion that RJV is more profitable for the insiders than NA. (Another feature of Figure 1 – the fact that $x_{IN}^{RJV}$ is increasing in $\omega$, so $x_{IN}^{RJV} > x_{NA}$ – is not quite so robust. Specifically, if $\gamma$ is sufficiently large, then we get $x_{IN}^{RJV} < x_{NA}$ when both $\theta$ and $\omega$ are strictly positive.\(^{28}\))

Finally, note that we have assumed that firms’ non-R&D fixed costs are independent of the NA/HM/RJV choice. However, this might not be the case. If, for example, there were even an infinitesimal fixed cost of implementing a horizontal agreement (either RJV or HM), then NA would dominate RJV if $\omega$ is sufficiently small.\(^{29}\) On the contrary, of course, both RJV and (especially) HM might present opportunities for savings in non-R&D fixed costs through the avoidance of duplication (e.g. the sharing of “back office” facilities) and “synergies”. Such savings are a further reason

---

\(^{28}\)Intuitively, large $\gamma$ implies that levels of R&D investment will be generally low throughout the industry. This means that the production of an RJV insider is similar in size to that of a firm under NA. Therefore, the marginal variable profit of R&D to an RJV insider will be lower than that to a firm in NA, so $x_{NA} > x_{IN}^{RJV}$. This occurs because the marginal variable profit of R&D depends both on the level of output and on how output changes with R&D (i.e. $\partial q_i^2 / \partial x_i = 2q_i \cdot \partial q_i / \partial x_i$), and the latter is greater under NA than for an RJV insider, who is committed to sharing additional R&D results with a product-market competitor (this is the “competitive-advantage externality” in Kamien *et al.*, 1992).

\(^{29}\)Moreover, since, as we show in the next section, (large $\beta$, small $\omega$) implies that RJV profit-dominates HM as long as $\theta$ is not too large, this shows how (with small $\omega$ and $\theta$, and large $\beta$) NA might be chosen in equilibrium in the presence of fixed costs of establishing a horizontal agreement.
for dropping NA from the equilibrium analysis.

The result that RJV is always more profitable for the insiders than NA greatly simplifies our analysis by allowing us to drop NA. The equilibrium choice is now between HM and RJV.

4.2 HM versus RJV: the central trade-off

Here we examine the insiders’ choice between RJV and HM. Given that RJV always dominates NA (as we showed in the previous section), the RJV/HM choice is also the equilibrium choice.

Davidson and Ferrett (2007) examine the insiders’ choice between HM and NA. HM allows the exploitation of R&D complementarities, but the reaction that it provokes from outsiders on the product market works to undermine its profitability (as noted by Salant et al., 1983). Therefore, a profitability trade-off between HM and NA emerges: higher $\theta$ favours HM but higher $\beta$, which means stronger product-market interactions, favours NA.

In our model, RJV resembles NA in that, in both scenarios, all firms in the industry make both their R&D and their production decisions independently to maximise their own (expected) profits. However, unlike NA, RJV does allow some probability, $\omega$, that R&D complementarities will be exploited. This is what lies behind our finding, discussed above, that the insiders always prefer RJV to NA. (It thus follows that the size of the region in parameter space where HM dominates NA will be larger than that where HM dominates RJV.)

Figure 2 below depicts the choice between HM and RJV, and the way in which brand similarity, $\beta$, may be traded off against contract quality, $\omega$, to maintain indifference on the part of the insiders. The solid line in Figure 2 is the locus of points such that $\Pi_{HM}^{IN} = \Pi_{RJV}^{IN}$, and above (below) the locus the insiders’ preferred horizontal agreement is RJV (HM).
While HM offers a higher probability than RJV that R&D complementarities will be exploited, it tends to be less profitable, for a given vector of marginal costs across all firms, than RJV on the product market. In product-market competition, HM suffers in comparison with RJV for the same reason that it underperforms NA: the profitability of merger is undermined by the reactions of outsiders. Therefore, to maintain indifference between HM and RJV on the part of the two insiders, a higher $\beta$, which increases the exposure of the merged firm to the reactions of outsiders, must be counterbalanced by a lower $\omega$ (probability of contract enforceability), which reduces the attractiveness of RJV. This explains why the HM/RJV indifference locus in Figure 2 is downward sloping.

Figure 2 is drawn for a given value of R&D complementarities, $\theta$. In Figure 3 below, we vary the value of $\theta$, and we see that increasing $\theta$ shifts the insiders’ HM/RJV indifference locus outwards. Therefore, a rise in the degree of R&D complementarity makes HM “more likely” relative to RJV,
in the sense that the region in parameter space where HM is chosen grows in size. This result is intuitive: a rise in $\theta$ increases the relative attractiveness of merger as a form of horizontal agreement because (unlike RJV) it offers certainty that the now-more-significant R&D complementarities will be successfully exploited. However, note that, even for “perfect” R&D complementarity (i.e. $\theta = 1$), it is still possible for the insiders to prefer RJV to HM despite $\omega < 1$. Typically, this requires high $\beta$, which (as we discussed above) undermines the profitability of HM.

FIGURE 3: Greater R&D complementarities make merger more likely (HM is chosen inside each indifference locus, and RJV outside)

Finally, we return to Figure 2. The HM/RJV indifference locus is relatively steep, and this implies: firstly, that the insiders may optimally choose HM even when $\omega = 1$ (this requires low $\beta$); and secondly, that the insiders may optimally choose RJV even when $\omega \approx 0$ (this requires high $\beta$).\footnote{We interpret $\theta = 1$ as perfect R&D complementarity in the sense that it implies that there is zero duplication between the insiders’ R&D activities.}

\footnote{Recall that the insiders are indifferent between RJV and NA when $\omega = 0$.}
Consider first the result that the insiders may choose merger even when RJV contracts are always enforceable. To see why, assume that $\beta = 0$, so that the brands are independent in demand and there are no strategic interactions on the product market. Given our earlier discussion of how product-market competition reduces the profitability of merger, one might have thought that an RJV and a merger would be equivalent with $\beta = 0$ and $\omega = 1$. However, in fact, HM strictly dominates RJV in this case (if $\theta > 0$): the insiders in the merged firm invest more in R&D and earn greater profits than the RJV insiders. This occurs because merger leads to the internalisation of an externality: one insider’s R&D spending benefits the other insider,\(^{32}\) but this external benefit is only taken into account in R&D determination under merger, where the insiders’ R&D investments are set to maximise joint profits. (In contrast, an RJV insider sets her level of R&D investment to maximise her own profits only.)

Our finding that, even with a negligible probability of contract enforceability, the insiders may prefer RJV to HM requires relatively similar brands, and it arises because the strong product-market interactions that accompany similar brands work to make HM unprofitable.

The key results from this section are listed as bullet points at the start of the conclusion in Section 6 below.

## 5 Illustrative Welfare Analysis

In this section we briefly report the salient findings of an exploratory numerical exercise that we have undertaken to investigate the model’s normative properties. Numerical values for profits and social welfare, which form the basis for our results, are reported in the Appendix. We use the following standard expression for social welfare (or surplus):

\[
W = a \sum_{1}^{N} q_i - \frac{1}{2} \sum_{1}^{N} q_i^2 - \beta \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} q_i q_j - \sum_{1}^{N} c_i q_i - \frac{\gamma}{2} \sum_{1}^{N} x_i^2 \tag{11}
\]

\(^{32}\)Kamien et al. (1992) call this the “combined-profits externality”.

19
This is merely utility minus production and R&D costs. In the NA equilibrium, where all firms are identical, social welfare is

\[ W_{NA} = N (a - c + x_{NA}) q_{NA} - \frac{N}{2} [1 + \beta (N - 1)] q_{NA}^2 - N \gamma x_{NA}^2 \] (12)

The expression for social welfare in the HM game is more complicated because of the asymmetry between insiders and outsiders:

\[ W_{HM} = a (2q_{HM}^I + (N - 2) q_{HM}^O) - \frac{1}{2} \left[ 2 (q_{HM}^I)^2 + (N - 2) (q_{HM}^O)^2 \right] - \beta \left[ (q_{HM}^I)^2 + 2 (N - 2) q_{HM}^I q_{HM}^O + \frac{1}{2} (N - 3) (N - 2) (q_{HM}^O)^2 \right] - 2 \left[ c - (1 + \theta) x_{HM}^I \right] q_{HM}^I - (N - 2) (c - x_{HM}^O) q_{HM}^O - \frac{\gamma}{2} \left[ 2 (x_{HM}^I)^2 + (N - 2) (x_{HM}^O)^2 \right] \] (13)

and the social welfare expression for RJV can be derived by straightforward analogy.

Our first finding is that, socially, RJV always dominates NA; that is, \( W_{RJV} > W_{NA} \) for every set of parameter values. This holds regardless of the absence or presence of outside firms (i.e. in both the \( n = 2 \) and \( n = 5 \) cases). This finding is analogous to the result we derived in section 4.1 above on the ranking of insiders’ profits, and it is unsurprising because an RJV allows R&D complementarities to be exploited without reducing the number of independent firms on the product market.

Next, we compare the social-welfare performance of RJV and HM. Our results here also hold independently of the absence/presence of outside firms. First, we find that, for small degrees of R&D complementarity (\( \theta = 0.1, 0.35 \)), \( W_{RJV} > W_{HM} \). This conforms to a standard view amongst competition-policy practitioners: RJV is to be socially preferred to horizon-

\[ \text{The limits on the summations in the third term ensure that each pair of quantities is interacted only once.} \]

\[ \text{The single, minor exception to this finding is with } n = 5 \text{ and } \theta = 0.35. \text{ Here, } W_{RJV} \text{ is very slightly less than } W_{HM} \text{ if } \omega = 0.1. \]
tal merger because it preserves product-market competition.\footnote{There is, of course, a debate, into which we do not enter here, concerning the welfare standard that is used in practice in competition policy (e.g. social welfare, consumer surplus, or some mix of the two).} Second, for intermediate degrees of R&D complementarity ($\theta = 0.65$), we find that RJV is socially preferred to HM if and only if contract quality is high ($\omega = 0.9$). One interpretation of this result is that poor contract quality makes the gains from a horizontal agreement “merger-specific” (in the sense of Farrell and Shapiro, 2001).

Third, for substantial R&D complementarities ($\theta = 0.9$), we find that HM socially dominates RJV for all levels of contract quality. (Thus, HM can be socially preferred to RJV even when contracts are always enforceable – i.e. when the sharing of R&D results is not “merger-specific”.) This arises for the same reason that the insiders can strictly prefer HM to RJV when $\omega = 1$ (see Section 4.2): R&D investment by one insider creates a positive externality for the other insider, and this is only internalised under HM.

Finally, we compare the private incentives of the insiders with the social preference between HM and RJV. As one would expect, HM is always chosen in equilibrium when there are no outside firms ($n = 2$). Therefore, given the discussion of society’s merger/RJV preference above, we can conclude that HM is chosen “too often” relative to RJV in the absence of outsiders. This is because consumers tend to benefit from the competition between insiders in the RJV case.

With outside firms ($n = 5$), the insiders might choose to form an RJV in equilibrium.\footnote{Low R&D complementarities ($\theta = 0.1$) favour RJV, and high ones ($\theta = 0.65, 0.9$) favour HM. For intermediate R&D complementarities ($\theta = 0.35$), the insiders only form an RJV when contract quality is high ($\omega = 0.9$).} (The presence of outsiders on the product market works to undermine the profitability of bilateral merger under Cournot competition.) Although merger is chosen less frequently than in the absence of outsiders, it is still chosen “too often” from a social perspective.\footnote{There are only two cases where the insiders’ preferences diverge from society’s: ($\theta = 0.35, \omega = 0.5$) and ($\theta = 0.65, \omega = 0.9$). In both of these, society prefers an RJV but the insiders choose to merge.} However, it appears that the presence of outside firms works to align the preferences of the insiders with those of society. (We conjecture that this arises because the presence of outsiders weakens the insiders’ preference for merger, but it also strengthens society’s preference for merger since, in the majority of cases,
the outsiders prefer HM to RJV.)

A task for future work is to investigate the robustness of these suggestive welfare findings.\textsuperscript{38}

6 Conclusion and Discussion

Our key contribution has been to examine the choice between horizontal merger and RJV as alternative vehicles for exploiting R&D complementarities, whereas most existing contributions examine just one of those forms of horizontal agreement in isolation. An innovation in our modelling structure is that we have allowed for the possibility that RJV contracts might be unenforceable.

Our key positive findings are as follows:

- For the two insiders, RJV always dominates no-agreement, so some form of horizontal agreement (merger or RJV) always arises in equilibrium.

- Merger/RJV tradeoff: While merger offers certainty that R&D complementarities will be successfully exploited, it leads to an aggressive and profit-reducing reaction by outsiders on the product market. Rises in both brand similarity and contract quality favour RJV over merger.

- A rise in the degree of R&D complementarity makes merger “more likely” relative to RJV. However, RJV may continue to be chosen even with perfect R&D complementarity.

- Surprisingly, the two insiders may optimally choose to merge even when RJV contracts are always enforceable (this requires a low degree of brand similarity); and they may optimally choose to form an RJV even when the probability of contract enforceability is negligible (this requires a high degree of brand similarity).

\textsuperscript{38}In particular, one might add a social indifference locus to a diagram like Figure 2.
How do our results relate to the BP/ARCO case study that motivated our analysis? If we assume that crude oil is a relatively homogeneous product and that the degree of R&D complementarity is high (so that both $\beta$ and $\theta$ are close to 1), then we can see from Figure 3 that the insiders will, as BP and ARCO did, prefer merger to RJV when contract quality is low. Moreover, the welfare analysis in Section 5 is also relevant. There, we showed that society prefers merger to RJV when R&D complementarities are substantial but contract quality is low. Therefore, our results suggest that a merger with the characteristics of BP/ARCO might benefit both the insiders and society.

To close, we briefly consider three possible generalisations of our analysis. The first is to allow for Bertrand (price) competition on the product market. We conjecture that our qualitative results will not survive this generalisation. This is because, under Bertrand competition, horizontal mergers are generally profitable for market-power reasons alone. Therefore, given that merger offers a greater chance of realising R&D complementarities, it is difficult to see why an RJV would ever be formed in a Bertrand market.

The second avenue for generalisation involves considering additional vehicles for exploiting R&D complementarities. Given that the insiders intend to pool their R&D results, there are, in principle, four distinct ways in which they could manage their R&D and output decisions. Our analysis has considered two of these: RJV, where the insiders choose their R&D and production levels independently of each other to maximise their own profits; and horizontal merger, where the R&D and output decisions of inside brands are co-ordinated to maximise joint profits.

However, there are two other potential set-ups that the insiders could adopt. The first involves collusion in R&D but competition in outputs. Kamien et al. (1992) refer to this as “RJV cartelization”, and it might conceivably be achieved in two ways: the RJV insiders could contract all

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39 Although the numerical analysis in Section 5 was conducted for a moderate degree of brand similarity ($\beta = 0.5$), this finding continues to hold for $\beta$ close to 1.

40 Indeed, with substantial R&D complementarities, society can prefer merger to RJV even if contracts are always enforceable. The intuition here follows that given for the insiders towards the end of Section 4.2.

41 Low RJV contract quality makes the “efficiencies” merger-specific, which qualifies the usual preference of competition policy practitioners for RJV over merger.

42 See, for example, Deneckere and Davidson (1985).
of their R&D activities out to a third party with instructions to maximise joint profits; or they could merge and implement “divisionalization” (in the sense of Baye et al., 1996) on the product market. For our purposes, it is important to note that both of these approaches might well involve their own significant contractual problems. However, setting these to one side, we conjecture that such “RJV cartelization”, if successfully implemented, would always dominate our RJV option. Secondly, the insiders might compete in R&D but collude in outputs – the “semi-collusion” case of Fershtman and Gandal (1994), and Brod and Shivakumar (1999). In the Cournot setting, where mergers for market-power reasons alone are generally unprofitable, we conjecture that the insiders will prefer our RJV option to such “semi-collusion”.

Thirdly, we could allow for endogenous RJV and merger size, rather than restricting horizontal agreements to just two participants. In comparison with the no-agreement outcome, where all firms remain independent, it seems likely that, if legally permitted, an RJV would expand to include the entire industry. However, it is unclear whether the same is true of a horizontal merger of variable size (because merger tends to make the insiders less aggressive on the product market, which benefits outsiders). Thus, it is possible that the optimal RJV and the optimal merger would include different numbers of insiders.

The systematic investigation of these conjectures remains a task for future work.

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43For example, if the RJV insiders contract out their R&D work, then the third party will require ongoing access to their accumulated R&D stocks and technical expertise (see Bhattacharya et al. (1992) for an early analysis). This could well present similar contractual complications to those we analyse. The key point (and the key contrast with merger) is that, within an RJV, firms remain independent entities whose ultimate concern is with their own profits.
References


Appendix: Numerical values for the illustrative welfare analysis in Section 5

All values are reported to three decimal places

Maintained values: $a = 10$, $\beta = 0.5$, $c = 5$, $\gamma = 4$

No outsiders ($n = 2$)

$W_{NA} = 16.117$

$\Pi_{NA} = 4.299$

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Outsiders present \((n = 5)\)

\[
W_{NA} = 21.178 \\
\Pi_{NA} = 1.509
\]

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