Okun’s law – A meta analysis

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Abstract

This paper seeks to identify whether there is a representative empirical Okun’s Law coefficient (OLC) and to measure its size. We carry out a meta regression analysis on a sample of 269 estimates of the OLC to uncover reasons for differences in empirical results and to estimate the ‘true’ OLC. On statistical (and other) grounds, we find it appropriate to investigate two separate subsamples, using respectively (some measure of) unemployment or output as dependent variable.

Our results can be summarized as follows. First, there is evidence of type II publication bias in both sub-samples, but a type I bias is present only among the papers using some measure of unemployment as the dependent variable. Second, after correction for publication bias, authentic and statistically significant OLC effects are present in both sub-samples. Third, bias-corrected estimated true OLCs are significantly lower (in absolute value) with models using some measure of unemployment as the dependent variable. Using a bivariate MRA approach, the estimated true effects are -0.25 for the unemployment sub-sample and -0.61 for the output-sub sample; with a multivariate MRA methodology, the estimated true effects are -0.40 and -1.02 for the unemployment and the output-sub samples respectively.

1 INTRODUCTION

Since the pioneering work of Okun (1962) and his famous result that a 3% increase in output is associated with a 1% decline in the rate of unemployment, a large stream of literature has been devoted to the so-called Okun’s Law, the responsiveness of the unemployment rate to
real output variations. As the Okun’s Law coefficient (OLC hereafter) continues to be a central parameter in the field of short run macroeconomics, it is not surprising that the empirical component of this literature has reported a proliferation of estimates of the correlation between unemployment and real GDP movements.

To date, however, no consensus has been reached regarding the size of the OLC. While there is only one Okun’s Law, several alternative theoretical models and empirical strategies have been used for estimating the value of the OLC. However, empirical estimates are often sensitive to model specification and particularly to whether output or unemployment is used as the dependent variable.

Other forms of differences in model specification arise from the choice about use of a static or dynamic model; and from the choice about use of first-difference (with output and unemployment variables expressed in first differences) or gap model (with output and unemployment variables expressed in terms of the cyclical components or deviations from long-term trends). In the case of the gap model, empirical results may also be sensitive to the choice of the detrending method (linear trend, HP filter, etc.).

While this literature is characterized by a diversity of models and empirical strategies and by a striking heterogeneity of empirical results, no systematic survey has been done. This diversity of models, empirical strategies, and results makes it difficult to use these estimated OLC values for the practical analysis of short run macro fluctuations. Moreover, as suggested by DeLong and Lan (1992), publication bias can be found in several fields of economic research and may thus potentially concern empirical analysis of the Okun relationship. Such bias will exist if the process of research publishing predominantly selects papers with statistically significant results. Hence, larger and more significant effects will be over represented while studies with small insignificant effects will be under represented or won’t be published. With publication selection, the average of effect magnitudes across papers may thus expected to be upwardly biased and the presence of large empirical effects in the literature is not statistically well-founded (Stanley 2005). Without correction for publication bias, it is thus not valid to take summary statistics of large empirical effects found the literature as indicative of true population values of the effect in question.
If the Okun’s Law literature has been subject to publication selection bias, averages of OLC estimates across papers are likely to be upwardly biased in magnitude (in absolute value) and so will be invalid as evaluations of the true value of the OLC. Economists have already tried to use meta regression analysis to test for publication selection and then to remove or lessen its effects (beginning with Stanley and Jarrel, 1989). This is precisely the main aim of this paper.

We undertake a meta regression analysis (MRA hereafter), to study whether the observed variation in OLC may be partly accounted for by the existence of such publication biases\(^1\). To the best of our knowledge, this is the first paper which performs a meta regression on Okun’s law. As the Okun’s Law is widely used as a rule of thumb for assessing the expected level of the unemployment rate, it is necessary to test whether there is reliable evidence of a genuine OLC after eliminating potential publication bias.

We then perform a multivariate MRA by including “moderator” dummy variables in an attempt to establish whether variations in OLC across studies are mainly due to data characteristics or to different model specifications. As the choice of real output or unemployment as dependent variable is a notable aspect of heterogeneous specifications in the empirical literature on the Okun’s Law, this choice may be expected to influence empirical estimates of the OLC (except if there were one cointegrating relationship between unemployment and real output, which is not found in the literature). Hence, we will investigate the influence of this specification choice by running separate investigations for the subsample of studies using real output as the dependent variable and for the subset of papers using unemployment as the endogenous variable.

Our results can be summarized as follows. First, there is evidence of type II bias in both sub-samples, but a type I bias is present only among the papers using some measure of unemployment as the dependent variable. Second, after correction for publication bias, authentic and statistically significant OLC effects are present in both sub-samples. Third, bias-corrected estimated true OLCs are significantly lower (in absolute value) with models using some measure of unemployment as the dependent variable. Using a bivariate MRA approach, the estimated true effects are -0.25 and -0.61 for the unemployment sub-sample and

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\(^1\) While meta analyses are often used in the field of medicine with independent individual studies, empirical studies on the Okun’s Law sometimes use non independent data sets including for example the US unemployment rate. However, as the starting and ending periods of the data base, together with the data frequency or the transformation of variables vary a lot across studies, the finally estimated results of these studies may be reasonably considered as independent from each other and included in a meta analysis.
the output-sub sample respectively; with a multivariate MRA methodology, the estimated true
effects are -0.40 and -1.02 for the unemployment and the output-sub samples respectively.

The paper is structured as follows. Section 2 briefly reviews the main issues in the empirical
research on the Okun’s Law. Section 3 describes the properties of the literature sample used
for the meta analysis. Section 4 explains our approach to implementing the MRA. Section 5,
using graphical analysis and bivariate MRA, tests for the existence and magnitude of
publication bias. This permits the authors to estimate (one or more) “authentic” Okun’s Law
coefficient beyond publication bias. The corresponding multivariate MRA is conducted in
Section 6. Section 7 concludes.

2. THEORETICAL BACKGROUND

Since Okun’s (1962) seminal paper, Okun’s law has widely been accepted in the literature as
a representation of the negative relation between unemployment and output. In his 1962
article, Okun presented two simple equations connecting the rate of unemployment to real
output which have frequently been used as rules of thumb for applied macroeconomic
analysis. Since that time, these equations have been expanded on and modified by many
authors so as to improve statistical fit and to make their theoretical foundation more precise.

A first group of papers includes two classes of specification suggested by Okun (1970): the
first difference model and the “gap” model. According to the first-difference model, the
relationship between the natural log of observed real output \( y_t \) and the observed
unemployment rate \( u_t \) is given by the expression

\[
\Delta u_t = a_0 + a_1 \Delta y_t + \epsilon_t
\]

(1)

where \( a_0 \) is the intercept, \( a_1 (a_1 < 0) \) is Okun’s coefficient measuring by how much changes
in output produce changes in the unemployment rate, and \( \epsilon \) is the disturbance term.

From the point of view of the “gap” model, the specification is given by the expression

\[
u_t - u^*_t = b_0 + b_1 (y_t - y^*_t) + \epsilon_t
\]

(2)
where $y^*$ represents the log of potential output, $u^*$ is the natural rate of unemployment and the other symbols have the same meaning as in equation (1). In this second specification, the left-hand side term represents the unemployment gap, whereas $(y_t - y_t^*)$ captures the output gap. In other words, the difference between the observed and potential real GDP captures the cyclical level of output. Likewise, the difference between the observed and natural rate of unemployment represents the cyclical rate of unemployment.

A major problem with the gap model is that there are no observable data on $y^*$ and $u^*$ so they have to be estimated. While Okun retained $u_t^* = 4\%$ as a target rate of labour utilization and favored a simple time trend to measure $y_t^*$, alternative time series approaches have been proposed in the literature for estimating $y_t^*$ and $u_t^*$. Among others, deterministic methods such as the Hodrick-Prescott filter (see for instance Marinkov and Geldenhuys 2007, or Moosa 2008) or the Baxter-King filter (see for instance Villaverde and Maza 2009) have been widely used while some authors selected stochastic decomposition procedures such as Beveridge and Nelson (see for instance Lee, 2000) or the unobserved component model suggested by Harvey (1989) and estimated with a Kalman filter algorithm (see for instance Moosa 1997, or Silvapulle et al. 2004). Finally, some papers use a specific auxiliary model to estimate these equilibrium values (see for instance Prachowny 1993, or Marinkov-Geldenhuys 2007).

As Okun noted that one of the shortcomings of the proposed relationship lies in the fact that the unemployment rate may only be considered as a proxy variable for idle resources affecting output losses, a second group of papers built empirical versions of the Okun’s Law from a macroeconomic production function relating real output to a set of factors potentially including labour, capital, and technology (see for instance Gordon, 1984). Assuming that the equilibrium real output is obtained when all the factors reach their equilibrium level, the production function can then be transformed into a gap version of Okun’s Law including the idle resources coming from each input and which can be written as:

$$y_t - y_t^* = c_0 + c_1(u_t - u_t^*) + c_2(Z_t - Z_t^*) + \epsilon_t$$

(3)

where $(Z_t - Z_t^*)$ is a vector of gaps between equilibrium and observed values of inputs different from labour. It is important to note that this kind of production function-version of
the Okun’s Law is then estimated with real output as the dependent variable instead of the unemployment rate.

Theoretically and econometrically, this reversal of the functional form of the estimated relationship makes it difficult to compare the empirical results found with two groups of studies: one group in which the first difference or the gap model in which unemployment change or gap is the dependent variable; the other obtained using the production function version of the Okun’s Law. It is well-known that the coefficient of a regression of X on Y is not in general equal to that in the inverse of a regression of Y on X. However, to make both groups of OLC estimates interpretable as the sensitivity of unemployment to real output changes, and so to facilitate comparison across the two groups of studies, coefficients estimated with equations using real output as the endogenous variable were systematically inverted, thereby rewriting all OLC values as the effect of real output variations on unemployment movements.

3. META ANALYSIS: LITERATURE SAMPLING

Here we describe the procedure retained for literature sampling for the meta regression analysis. In order to select a sample of OLC empirical studies which is both representative of this literature and of a manageable size, we have resorted to a structural search for articles using the following sampling criteria. First, we searched the EconLit database for empirical studies on the OLC and all the papers that fulfilled the following criteria have been selected: (i) key words used in the search were: “Okun’s Law” and “Output-unemployment relationship”; (ii) an abstract is presented so that the presence of econometric estimations of the OLC can be checked; (iii) the article was published after 1980 and was listed in the EconLit database as of December 2010.

1980 was retained as the starting date in order to permit analysis of the variance of published OLC empirical estimates but within relatively unified econometric frameworks and with data sets of the same quality and with reasonable time length. Dynamic time series methods with regards to data transformation, data stationarity, and optimal lag selection became increasingly common in the eighties. Prior to 1980, many papers used very short data series (for instance, Thirlwall, 1969, used annual data from 950 to 1967 with just 18 data points) or statistically-questionable methods (such as empirically estimated time trends or ad hoc
coefficients in order to calculate potential output or the natural rate of unemployment). All papers not related to the research question have been excluded. This selection process identified 97 papers.

After having examined these 97 articles, we excluded studies that do not include any original econometric estimation of the Okun’s Law coefficient. We also excluded studies that do not give sufficient information concerning the type of estimated model (endogenous/exogenous variables), the data base (initial and final dates, periodicity) or the empirical results (R-squared value, estimated coefficients and standard errors). We decided to exclude studies including only non linear Okun’s Law models. Finally, it is important to note that while some studies suggest that Okun’s law has undergone structural change over time (e.g. Lee (2000), Huang and Chang (2005), Sögner and Stiassny (2002)), over countries (Kaufmann (1988), Lee (2000), Moosa (1997)) or over the course of the business cycle (e.g. Crespo-Cuaresma (2003), Huang and Chang (2005), Silvapulle et. al (2004)), we decided to restrict our data base to linear versions of the Okun’s relationship assumed to be stable across the whole data sample. This choice was motivated by the following reasons. First, these studies predominantly use either non linear models such as threshold models which include ad hoc assumptions concerning the threshold variable (the previous level of unemployment or the previous growth rates of real output for instance) or time varying models where empirical results may appear highly dependent upon the characteristics of the retained methodology (the size of the rolling window, for example). Incorporating these papers in the data base would thus go in hand with a large increase of the set of conditioning variables in the multivariate meta regression model with a limited number of observations associated with each variable. Second, due to the sensitivity of the estimated results to the retained testing procedure, these papers often lead to heterogeneous results and may give rise to controversies (see for instance the recent debate between Owyang and Sekhosyan (2012) and Ball, Leigh and Loungani (2012) on the stability of the Okun’s Law relationship during the Great Recession).

As a consequence, while the comparison of the empirical results produced by linear and nonlinear models within a meta regression analysis may constitute an interesting area of research, it seemed a priori difficult to include both linear model and heterogeneous non linear models within the same meta regression sample. The total number of studies left after applying these criteria was 28 and the total number of observations in our database is 269,
each corresponding to one regression. Figure 1 shows the “life cycle” of this literature in terms of the number of documents recorded in EconLit and retained in the present MRA.

![Figure 1: The number of retained EconLit publications on the OLC](image)

As can be seen, the average number of papers meeting our selection criteria increased after 2003 and the literature peaked in 2007. Even base specifications of the Okun’s Law model permitted more than one regression per study since this specification is often applied to different samples, different time periods, and different measure of the output gap or of the variation of the unemployment rate around its equilibrium level. In accordance with common practice in meta regression analysis, these were recorded as independent regressions in order to investigate the influence of these heterogeneities on the published effect. The full list of studies included in the MRA is given in the list of References at the end of this paper (each being marked by an * symbol).

Table 1: Descriptive statistics of OLC studies (28 studies) and OLC estimates (269 estimators)

<table>
<thead>
<tr>
<th></th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLC</td>
<td>-3.22</td>
<td>0.17</td>
<td>-0.77</td>
<td>0.71</td>
<td>-0.58</td>
</tr>
<tr>
<td>Number of observations</td>
<td>21</td>
<td>408</td>
<td>50.4</td>
<td>46.54</td>
<td>41</td>
</tr>
<tr>
<td>First year</td>
<td>1948</td>
<td>1990</td>
<td>1968.2</td>
<td>10.75</td>
<td>1970</td>
</tr>
<tr>
<td>Last year</td>
<td>1985</td>
<td>2006</td>
<td>1999.2</td>
<td>4.61</td>
<td>1999</td>
</tr>
</tbody>
</table>

Proportion of OLC estimators with the following features (%)
<table>
<thead>
<tr>
<th>Time series data base</th>
<th>98.9</th>
<th>Country</th>
<th>74.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel data base</td>
<td>1.1</td>
<td>Region</td>
<td>26.0</td>
</tr>
<tr>
<td>Yearly frequency</td>
<td>68.5</td>
<td>European countries</td>
<td>74.4</td>
</tr>
<tr>
<td>Frequency higher than year</td>
<td>31.5</td>
<td>Unites States</td>
<td>7.6</td>
</tr>
<tr>
<td>Endogenous variable : Unemployment rate</td>
<td>41.8</td>
<td>Rest of the world</td>
<td>18.0</td>
</tr>
<tr>
<td>Endogenous variable : Real output</td>
<td>58.2</td>
<td>Static model</td>
<td>53.6</td>
</tr>
<tr>
<td>Model in level</td>
<td>9.2</td>
<td>Dynamic model</td>
<td>40.0</td>
</tr>
<tr>
<td>Model in first difference</td>
<td>14.7</td>
<td>Cointegrated model</td>
<td>6.4</td>
</tr>
<tr>
<td>Equilibrium values of real output and unemployment from filtering procedure</td>
<td>76.1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1 presents salient characteristics of the papers retained for our MRA. The number of observations used in the OL equations varied enormously. The smallest was 21, while the largest number was 408. All but 1.1% of the OLC were estimated from time series data bases and more than half of the studies (68.5%) used annual frequency. Nearly three quarters of the papers use country level data while the remaining papers use regional data bases. The percentage of estimates obtained with either the gap or the first difference version of the OL equation (41.8%) is close to the percentage of estimates obtained with production function versions of the OL (58.2%).

4. THE META ANALYSIS FRAMEWORK: TESTING FOR PUBLICATION BIAS AND ESTIMATING THE TRUE COEFFICIENT

The process of academic publishing may influence the characteristics of the published results. While several kinds of publication biases can appear, two specific biases are most often encountered (Stanley, 2005). Type I bias occurs when editors, referees, and/or researchers have a preference for a particular direction of results. Positive estimates of the OLC, for instance, might be ignored as it is hardly interpretable that short run movements of unemployment are positively correlated with output gap fluctuations. However, even if there are very strong theoretical reasons for expecting negative estimates of the OLC, at least a few studies should report positive estimates. We can, for example, imagine the case of specific labour market regulations in case of macroeconomic downturns. A positive OLC finding may
also arise due to some characteristics of data sets or of empirical methodologies. Such a bias would make the average taken from the published literature larger (in absolute value) than the estimated true effect.

Type II bias arises when editors, referees, and/or researchers have a preference for results that are statistically significant. As smaller samples and limited degrees of freedom reduce the probability of finding a significant result, this kind of publication bias may appear when researchers using small samples are inclined to search across econometric “tools” (proxies, estimators, specifications) in order to produce more significant results. Type II selection will thus lead to excess variation (Stanley, 2005).

Detection of the presence of type I publication bias most commonly starts with the so-called funnel plot which compares the effect size for each regression (here the OLC) against some measure of its precision (the inverse standard error of the OLC, Egger at al. 1997). In the case of no bias, the plot should appear as an inverted funnel: observations with high precision should be concentrated closely to the true effect, while those with lower precision should be more spread at the base of the plot. In the absence of type I publication bias, the funnel plot is thus symmetric.

This visual investigation can also be supplemented with explicit regression tests. The funnel asymmetry test (FAT) due to Egger et al. (1997) is implemented by means of the regression:

\[ OLC_i = \alpha + \beta \cdot SE_i + u_i, \quad i = 1, \cdots, N \]  

where \( OLC_i \) is the \( i \)th estimate of the OLC, \( SE_i \) is the standard error of point estimate \( i \), \( N \) is the number of estimates of the OLC and \( u_i \) is the regression error term. In this simple MRA, \( \alpha \) denotes the true OLC, and \( \beta \) indicates the size of publication bias.

As regression (4) is heteroskedastic and the measure of heteroskedasticity is the standard error of the estimate of the OLC, Stanley (2008) suggests performing weighted least squares by dividing equation (4) by the standard error of the OLC. This is simply achieved by OLS estimation of the transformed regression equation:
\[
\frac{OLC_i}{SE_i} = t_i = \beta + \alpha \cdot \left( \frac{1}{SE_i} \right) + \nu_i, \ i = 1, \cdots, N
\]  
(5)

where \( t_i \) is the \( t \)-statistic measuring the significance of the \( i^{th} \) OLC. Equation (5) represents a regression line through a funnel graph which is rotated by 90 degrees and which is adjusted for heteroskedasticity. The FAT test for publication bias is then a simple \( t \)-test on the intercept of equation (5); a \( \beta \) significantly different from zero indicates the presence of publication bias. If \( \beta \) is significantly positive (or negative), then the effect size is subject to an upward (or downward) bias. Moreover, there is evidence of a “true” empirical effect (that is, a systematic relationship between unemployment variation and real output movements) if the coefficient \( \alpha \) is significantly non-zero.

As the process of selecting estimates from the literature makes meta-analysis highly vulnerable to data contamination, the robustness of this basic test is checked by re estimating equation (5) with the iteratively re-weighted least squares method (IRLS) as in Krassoil Peach and Stanley (2009) or Havranek (2010).

In a similar way to the case of the type I bias, a visual inspection for the presence of type II bias can be assessed using the Galbraith plot (Galbraith 1988). This consists of a scatter diagram of the precision of the estimates of the OLC against the \( t \)-statistics corresponding to those estimates for a given assumed value of the true effect. If there were type II selection, large values (in absolute terms) will be over reported and there will be an excessive likelihood of reporting significant results. In case there was no type II publication bias and the true effect (labeled TE) were really true, the statistics \( |(OLC_i - TE)/SE_i| \) should not exceed 2 more than 5% of the time and the cloud should be randomly distributed around 0, with no systematic relation to precision.

The method of testing for type I bias can also be used to test for significance of the true effect beyond publication bias. The precision effect test (PET) is a simple \( t \)-test on the slope coefficient \( \alpha \) of equation (5).

As one of the main objectives of most meta analyses is to determine the dependencies of empirical results on characteristics of empirical strategy and design, we finally use the general multivariate version of the FAT-PET method which is specified as follows:
\[
\frac{OLC_i}{SE_i} = t_i = \beta + \alpha \cdot \left( \frac{1}{SE_i} \right) + \sum_{k=1}^{K} \gamma_k \cdot \left( \frac{Z_{ki}}{SE_i} \right) + \omega_i , i = 1, \cdots, N
\]  

(6)

where \( Z_{ki} \), \( k = 1, \cdots, K \) are meta-independent variables assumed to potentially affect the estimate of the OLC and \( \omega_i \) is the meta regression disturbance term, which has the standard characteristics. Each of the \( Z_{ki} \) is weighted by \( (1/SE_i) \) and the \( \gamma_k \) are \( K \) coefficients to be estimated, where each one measures the impact of the corresponding variable on the OLC.

The meta-independent variables used in this paper are presented in Table 2. We focus on a set of variables constructed to represent the following characteristics of models used in the Okun’s law empirical literature. Regarding the influence of sample features on empirical results we concentrate on the initial and final dates (respectively FIRSTYEAR and LASTYEAR) of the studies; we distinguish between time series data (SAMPTS) and panel data (SAMPPA); between samples dealing with annual data (FREQY), semestrial or quarterly data (FREQSQ); between samples using country-level (COUNT) or regional-level (REG) data sets; and finally between papers that focus on OECD countries (OECDCOUNT) and papers centered on non OECD countries (NOECDCOUNT). While there may be variance across countries within each of the OECD and non OECD groups, these dummies control for a variety of institutional characteristics (such as property rights regimes and labour mobility conditions) that may differ systematically between, but not within, the two groups.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description of the variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIRSTYEAR</td>
<td>First year of the sample</td>
</tr>
<tr>
<td>LASTYEAR</td>
<td>Last year of the sample (do you want a space between LAST and YEAR?)</td>
</tr>
<tr>
<td>SAMPTS</td>
<td>Dummy, 1 if the study uses a time series data base, 0 otherwise</td>
</tr>
<tr>
<td>SAMPPA</td>
<td>Dummy, 1 if the study uses a panel data base, 0 otherwise</td>
</tr>
<tr>
<td>FREQY</td>
<td>Dummy, 1 if the study uses annual data, 0 otherwise</td>
</tr>
<tr>
<td>FREQSQ</td>
<td>Dummy, 1 if the study uses semestrial or quarterly data, 0 otherwise</td>
</tr>
<tr>
<td>COUNTDED</td>
<td>Dummy, 1 if the data base only includes developed countries, 0 otherwise</td>
</tr>
<tr>
<td>COUNTDING</td>
<td>Dummy, 1 if the data base only includes developing countries, 0 otherwise</td>
</tr>
<tr>
<td>COUNT</td>
<td>Dummy, 1 if the data base only includes countries, 0 otherwise</td>
</tr>
<tr>
<td>MODSTA</td>
<td>Dummy, 1 if the model is static, 0 otherwise</td>
</tr>
<tr>
<td>MODDYN</td>
<td>Dummy, 1 if the model is dynamic, 0 otherwise</td>
</tr>
<tr>
<td>OTHEXO</td>
<td>Dummy, 1 if the model includes other exogenous variables than the unemployment variable or the GDP variable, 0 otherwise</td>
</tr>
<tr>
<td>NOOTHEXO</td>
<td>Dummy, 1 if the model includes no other exogenous variables than the unemployment variable or the GDP variable, 0 otherwise</td>
</tr>
</tbody>
</table>
Regarding equation characteristics, as explained previously we first distinguish between models using unemployment as the endogenous variable (ENDU) and models using real output as the endogenous variable (ENDY). We then distinguish between static (MODSTA) and dynamic models (MODDYN), between models including only one exogenous variable (NOOTHEXO) and models including several additional exogenous variables (OTHEXO), and then between single equation models (NEQ1) and multi equations models (NEQN). As the empirical evaluation of potential output and natural unemployment are essential steps in the estimation of the OLC, we also tried to take into account the precise nature of the econometric procedure retained for estimating these two variables. We thus constructed separate dummies for distinguishing between a linear trend methodology (FILTLT), an HP filter (FILTHP), a Baxter-King Filter (FILTBK), a Beveridge-Nelson procedure (FILTBN), an unobserved component mode (FILTUC) or an explicit model such as a production function for potential output (FILTMOD). In order to investigate more deeply the influence of model characteristics, we also included separate dummies for distinguishing between models in levels (LEVEL) and models in first difference (DELTA).

5. GRAPHICAL INVESTIGATION AND BIVARIATE TESTING FOR PUBLICATION BIAS AND TRUE EMPIRICAL EFFECT
As it is now common in applied MRA, we start by investigating the presence of type I publication bias by using the funnel plot technique. Figure 2a and 2b display the funnel plots for the unemployment sub-sample and the real output sub-sample, respectively. As a measure of precision, we use the inverse of the standard deviation of point estimates, which is plotted on the vertical axis; estimates of the OLC are plotted on the horizontal axis.

There are no positive estimates in the real output sub-sample and only seven positive estimates in the unemployment subsample so that the plot is clearly overweighed on the left side in both cases. This asymmetry is strongly suggestive of publication bias. Even though macroeconomic theory generally leads to the prediction of a negative OLC, an unbiased set of empirical evidence on the OLC would be consistent with a symmetric distribution of estimated OLC around a negative mean. For the unemployment sub-sample, visual inspection suggests a somewhat bimodal distribution of estimates; the mean of the two most precisely estimated values places the top portion of the funnel around -0.10, although the average of the top five points on the chart is substantially larger in magnitude, at around – 0.3. In the case

![Figure 2a: Funnel plot (unemployment sub-sample)](image1)

![Figure 2b: Funnel plot (real output sub-sample)](image2)
of the real output sub-sample, the top portion of the funnel is close to -1.63 and the average of
the top five points on the graph equals -1.35. These top values are quite far from the average
of all the estimates (larger by 54% in the case of the unemployment sub-sample and lower by
98% in the case of the real output subsample). Although there is a very high probability that
the OLC is in fact negative, the potential magnitudes of the bias show that simple summaries
of this literature may lead to a biased evaluation of the true size of the OLC.

As visual inspection of the funnel plots can be misleading and vulnerable to subjective
interpretation, the funnel graphs are now supplemented with the FAT performed using
Equation (5). Table 3 summarizes FAT results for the same samples as discussed before.
Table 3: Tests of type I publication bias and the true effect

<table>
<thead>
<tr>
<th>Dependent variable = t-statistic on the OL coefficient</th>
<th>OLS estimator</th>
<th>IRLS estimator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obs.</td>
<td>$\beta$ (bias)</td>
<td>$\alpha$ (precision effect)</td>
</tr>
<tr>
<td>Output sub-sample</td>
<td>157</td>
<td>-2.163***</td>
</tr>
<tr>
<td>Unemployment sub-sample</td>
<td>112</td>
<td>0.171***</td>
</tr>
</tbody>
</table>

(a) Empirical results obtained with the sub sample studies using some measure of real output as the dependent variable are presented in the row labeled “Output sub-sample”, and empirical results obtained with the sub sample studies using some measure of unemployment as the dependent variable are presented in the row labeled “Unemployment sub-sample”. Values of the t-statistics are given in parentheses. *** indicates significance at the level of 1%.

Before performing the FAT tests on each sub-sample separately, we start by testing for the null that the data don’t need to be split into these two sub-samples. In order to do so, we merge the two subsamples then perform an OLS-estimation of equation (5) with the whole sample. We then perform a Chow test for the selected null hypothesis. The test produces an F statistic of 13.594 with an associated p value of 0.000 which clearly confirms the rejection of the null. As a result, the remaining part of the paper will in the main focus on these two subsamples separately.

We now consider the sign and significance of publication bias for each of the two sub-samples. First consider the sub-sample of studies with real output as the dependent variable (denoted “output sub-sample” in Table 3). Here, the estimated sign of $\beta$ suggests that the direction of a publication bias is negative. Moreover, using either OLS or IRLS estimator, the FAT test shows that the $\beta$ coefficient (intercept term) is highly significant, so that the null of no type I publication bias is strongly rejected. Also note that not only is the $\beta$ coefficient negative, but its size is larger than 2 in absolute value (or nearly 2 in the case of the IRLS estimator), which might be considered as an indication of a “severe selectivity” effect according to Doucouliagos-Stanley (2008).

The story is different for the case of the sub-sample of studies with the unemployment rate as dependent variable (denoted “Unemployment sub-sample” in Table 3). In this case, the $\beta$ coefficient is positive, but its size is smaller than 2 in absolute value (or nearly 2 in the case of the IRLS estimator), which might be considered as an indication of a “mild selectivity” effect according to Doucouliagos-Stanley (2008).

For the combined (whole) sample, the estimated bias is negative.
coefficient is not significant with both OLS and IRLS estimators, so that the hypothesis of no type I publication bias is not rejected in this sub sample.

Hence we find that a type I bias is present only in the sub sample of papers estimating the Okun’s Law coefficient with empirical models using real output as the dependent variable. The difference between studies using real output as the endogenous variable and studies using unemployment rate as the endogenous variable is an important finding: while the first group of papers seems to be plagued by publication bias, the null hypothesis that the second group is not affected by this problem cannot be rejected at the usual confidence level.

We now turn to type II bias, and begin by examining the Galbraith plots shown in Figure 3a and 3b for the output sub-sample and the unemployment subsample respectively (the horizontal lines are the +2 and -2 limits for the t-statistics). The reported t-statistics exhibit both a wide variation and an apparent tendency to decline with rising precision. This visual
examination of the Galbraith plots can be complemented by the use of $z$-type tests on the proportion of significant t-statistics. Table 4 reports the results of these $z$-tests.

<table>
<thead>
<tr>
<th>Endogenous : Real output</th>
<th>Proportion of Significant t-stat$^{(a)}$</th>
<th>Z</th>
<th>P.value</th>
<th>Assumed True Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>84%</td>
<td>41.50</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>60%</td>
<td>30.66</td>
<td>0.00</td>
<td>-1.60$^{(b)}$</td>
</tr>
<tr>
<td>Endogenous : Unemployment</td>
<td>76%</td>
<td>38.80</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>65%</td>
<td>34.95</td>
<td>0.00</td>
<td>-0.275$^{(b)}$</td>
</tr>
</tbody>
</table>

(a) Significance at the 5% confidence level  
(b) True effect evaluated from the top 10% of the corresponding funnel graph

As can be seen in the Galbraith plots for the output sub-sample and the unemployment subsample, type II biases seem to be present in both of these two sub samples. Assuming that there is no underlying true effect ($TE = 0$), only 5% of the studies should report t-statistics larger than 2. However, the proportions of studies reporting t-statistics exceeding 2 are close to 84% and 76% respectively and the null hypothesis that the proportion of significant t-statistic is equal to 5% is systematically rejected when the TE is taken to be zero ($z = 41.50$ with $p < 0.0000$ for the output sub-sample and $z = 38.80$ with $p < 0.0000$ for the unemployment sub-sample). Moreover, implementing the tests for a value of the TE evaluated from the top 10% of the corresponding funnel graphs, the null hypothesis that the proportion of significant t-statistic is equal to 5% is again strongly rejected ($z = 30.66$ with $p < 0.0000$ for the output sub-sample and $TE = -1.601$ and $z = 34.95$ with $p < 0.0000$ for the unemployment sub-sample and $TE = -0.275$).

While studies using real output as the endogenous variable and studies using unemployment rate as the endogenous variable exhibited different results with respect to the null hypothesis of no type I publication bias, the null of no type II bias is now rejected for both sub samples (and also for the combined, whole sample as it happens) In the literature on the OLC, this excess variation may thus reflect selection for statistically significant results.
Whereas the detection of the presence of publication bias is a necessary step in analyzing the literature, a more important question concerns whether there is an underlying true effect, irrespective of publication selection (the so-called “true effect”). As suggested by Stanley (2008), Equation (5) may also be used to test for an authentic empirical effect beyond publication bias. Empirical results of performing the PET on the slope coefficient $\alpha$ of equation (5) highlight the following points.

Using the $\alpha$ (precision effect) point estimates and t statistics reported in Table 3, the 95% confidence intervals reported by PET for the unemployment rate sub-sample are: [-0.33 ; -0.20] with OLS and [-0.41 ; -0.09] with IRLS. In the case of the output sub-sample, empirical estimates of the TE are much larger (in absolute values) since they vary from [-0.72 ; -0.52] with OLS to [-0.70 ; -0.50] with IRLS. Note that simulations performed by Stanley (2008) show that taking the average of the PET estimate of the TE $\alpha$ in equation (5) and of the simple average of OLC estimates reported in the literature may greatly improve the accuracy of the TE estimate in case of publication selection. Here, calculating this average with both the OLS and IRLS estimators of $\alpha$, leads to (absolute) TE values close to 0.795 for the output-subsample and 0.265 with the unemployment rate sub-sample. Aside from the evident sensitivity of results to the estimation procedure, the TE obtained for the OLC appears to be systematically larger (in absolute value) for the output sub-sample than for the unemployment sample. Empirical models aimed at estimating the OLC by using models specified with real output as the dependent variable thus seem to lead to large estimators of the sensitivity of unemployment movements to real output fluctuations.

6. MULTIVARIATE META-REGRESSION ANALYSIS

To implement multivariate the MRA, we estimate equation (6) for each sub-sample successively. Each regression initially includes all the dummy explanatory variables listed in Table 2, other than those which have to be omitted so as to avoid linear dependence (in which case the constant term represents the effects of the omitted dummies). In this paper, the omitted dummies are SAMPTS, FREQY, COUNT, COUNTDED, MODSTA, NOOTHEXO, NEQ1, and DELTA.
Each model is first estimated with OLS. Insignificant variable are then excluded with a stepwise procedure involving both specific to general (or forward) and general to specific (or backward) selection steps to specify the finally estimated model. More precisely, variables are added to the model sequentially until no variable not yet in the model would, when added, have a t-statistic with a p value smaller than 0.05. Each time a variable is added to the model, variables with the lowest t-statistics are deleted until all remaining variables have a p value smaller than 0.05.

A robustness check was then performed by re-estimating the finally retained model with the iteratively re-weighted least squares method (IRLS) procedure. Meta-explanatory variables that appear as significant with both OLS and IRLS estimation of the finally selected model can be considered as the most influential effects on the value of the OLC. Lastly, in order to take into account the fact that the so-called “economics research cycle” (Havranek, 2010) may influence the size of the OLC, the year of publication (YEAR) and its square (YEAR2) are also added to the list of the finally selected significant variables. According to the economics research cycle hypothesis, when pioneering empirical results are published, they are often quickly confirmed by other publications exhibiting highly significant estimates. After that, publishing skeptical results or empirical results that diverge with initial results may become preferable for editors in order to feed the controversies. A positive coefficient associated with the variable YEAR and a negative coefficient associated with YEAR2 (with joint significance) may indicate that the economics research cycle hypothesis is consistent with the data at hand in fully specified models. Empirical results are reported in Table 5.

In order to obtain more information about the influence of the endogenous variable on the OLC estimates, equation (6) is first estimated for the whole sample with the whole set of explanatory variables, including ENDY (which equals 1 if real GDP is used as the endogenous variable and 0 otherwise). In this case, the constant term captures the influence of omitted variables for the sub-sample of models with unemployment rate as the endogenous variable and the coefficient associated with the dummy ENDY, where it is non-zero and significant, indicates by how much the OLC changes when moving from the unemployment sub-sample to the real output sub-sample.

This initial regression including the whole set of moderator variables is presented in the first two columns of Table 5. The last four columns in Table 5 present the empirical results for the
unemployment subsample and the output subsample respectively. For each pair of columns in the table, the first column in the pair lists unrestricted OLS regression results, while the second reports results from the IRLS estimator after applying the stepwise testing down procedure.

For the whole sample and each of the two sub-samples, F tests indicate that the estimated coefficients are jointly significant. However, in the unrestricted regressions, low values of t statistics indicate that some coefficients may be non-significant. This is confirmed by the stepwise testing down procedure.

For the whole sample, the results of the multivariate analysis are consistent with the bivariate FAT model and also suggest the presence of a publication bias. Moreover, the estimated true OLC equals -0.53 (with 95% confidence interval (-0.64, -0.42)) with the IRLS procedure. Note that in this multivariate analysis, the coefficient of the precision effect can be considered as a measure of the OLC for studies corresponding to the omitted dummies (i.e. studies using annual time series data for developed countries and single equation models specified as static relationships involving the first difference of unemployment rate as the dependent variable and the first difference of real output as the only dependent variable). As suggested by the significance of the coefficient associated with the moderator variable ENDY, studies using a model specified with output as the dependent variable tend to yield larger absolute values of the OLC (a positive sign means that the value of the OLC increases towards zero while a negative sign means that the value of the OLC decreases away from zero). Moreover this effect appears to be highly significant, as revealed by the associated t-statistics. The use of real output instead of the unemployment rate as the dependent variable on the Okun’s Law equation specification increases the absolute value of the OLC by 0.390 (on average). As the estimated OLC in the sample are harmonized so as to represent the impact of output on unemployment, the coefficient of unemployment retained for this group of studies is simply the inverse of the coefficient associated with unemployment (or employment) in the real output equation. As a consequence, the large negative values of the OLC estimated in this group of studies may result from the fact that estimating some form of production function leads to an underestimation of the sensitivity of output to employment (or unemployment) because of simultaneity bias. The OLC calculated as the inverse of this coefficient is thus mechanically overestimated.
Table 5: Multivariate meta regression analysis

<table>
<thead>
<tr>
<th></th>
<th>Whole sample</th>
<th>Unemployment sub-sample</th>
<th>Output sub-sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS</td>
<td>STEPWISE then IRLS</td>
<td>OLS</td>
</tr>
<tr>
<td>Constant</td>
<td>-240.41 (-2.01)</td>
<td>-194.45 (-3.00)</td>
<td>-286.50 (-0.72)</td>
</tr>
<tr>
<td>Precision</td>
<td>-0.400 (-3.08)</td>
<td>-0.528 (-9.44)</td>
<td>-0.289 (-1.15)</td>
</tr>
<tr>
<td>SAMPPA</td>
<td>-0.261 (-1.74)</td>
<td>-0.174 (-1.80)</td>
<td>-0.289 (-1.15)</td>
</tr>
<tr>
<td>FRENSY</td>
<td>0.152 (1.37)</td>
<td>0.186 (4.38)</td>
<td>0.147 (0.72)</td>
</tr>
<tr>
<td>COUNTDING</td>
<td>0.188 (3.83)</td>
<td>0.225 (4.83)</td>
<td>0.139 (1.65)</td>
</tr>
<tr>
<td>REG</td>
<td>0.334 (2.67)</td>
<td>0.293 (3.71)</td>
<td>0.139 (1.65)</td>
</tr>
<tr>
<td>MODDYN</td>
<td>0.117 (2.36)</td>
<td>0.145 (2.96)</td>
<td>0.008 (0.09)</td>
</tr>
<tr>
<td>OTHSEX</td>
<td>0.138 (2.16)</td>
<td>0.218 (5.54)</td>
<td>0.012 (0.10)</td>
</tr>
<tr>
<td>NEQN</td>
<td>-0.057 (-1.65)</td>
<td>-0.071 (-1.39)</td>
<td>1.371 (5.33)</td>
</tr>
<tr>
<td>ENDY</td>
<td>-0.437 (-3.35)</td>
<td>-0.390 (-6.22)</td>
<td>-0.253 (-1.89)</td>
</tr>
<tr>
<td>LEVEL</td>
<td>-0.124 (-1.71)</td>
<td>-0.211 (-5.85)</td>
<td>1.371 (5.33)</td>
</tr>
<tr>
<td>FILLT</td>
<td>-0.153 (-1.09)</td>
<td>-0.055 (-0.11)</td>
<td>1.371 (5.33)</td>
</tr>
<tr>
<td>FILTHP</td>
<td>-0.031 (-0.54)</td>
<td>-0.008 (-0.08)</td>
<td>1.371 (5.33)</td>
</tr>
<tr>
<td>FILTBN</td>
<td>-0.160 (-1.00)</td>
<td>0.022 (0.05)</td>
<td>1.371 (5.33)</td>
</tr>
<tr>
<td>FILTUC</td>
<td>-0.300 (-1.20)</td>
<td>-0.325 (-0.72)</td>
<td>1.371 (5.33)</td>
</tr>
<tr>
<td>FILTMOD</td>
<td>-0.019 (-0.16)</td>
<td>-0.012 (-0.05)</td>
<td>1.371 (5.33)</td>
</tr>
<tr>
<td>AVGYEAR</td>
<td>0.545 (0.88)</td>
<td>0.120 (1.99)</td>
<td>0.138 (3.22)</td>
</tr>
</tbody>
</table>

R2                | 0.65         | 0.61                    | 0.62              | 0.57             |
F-test (P. val.)  | 27.036 (0.00) | 159.347 (0.00)          | 27.921 (0.00)     | 176.800 (0.00)   |
Reset test (P. val.) | 0.061 (0.80) | 0.024 (0.87)           | 0.003 (0.95)      | 1.631 (0.20)    |

For each estimated coefficient, the corresponding t-statistic is indicated in parentheses. The F-statistic tests the null hypothesis that independent variables are jointly equal to zero. The Ramsey Reset test corresponds to the null hypothesis of no omitted variable (linear functional form).

When splitting the whole sample so as to analyze separately the group of studies involving an Okun’s Law model with unemployment rate as the endogenous variable and the group of studies with real output as the endogenous variable, the multivariate models lead to empirical results for publication bias and authentic empirical effect which are fully consistent with those from bivariate MRA. Papers with real output as the endogenous variable are affected by negative publication bias while no publication bias appeared as statistically significant in the case of papers with unemployment rate as the endogenous variable. Moreover, authentic
empirical effects are significant in both groups of papers with a lower value (in absolute terms) for the group of studies with unemployment rate as the endogenous variable. The precision effect equals \(-0.40\) (with 95\% confidence interval \([-0.47, -0.34]\)) for the unemployment subsample and \(-1.02\) (with 95\% confidence interval \([-1.15, -0.88]\)) for the output sub-sample.

For both sub-samples, it is important to note that the influence of the filtering procedure (such as the HP filter, or the Baxter King filter or Beveridge Nelson filter) is never significant after selection of the most influential moderator variables with the stepwise methodology. Finally, as in the case of the bivariate MRA, the hypothesis of an “economics research cycle” is systematically rejected at the 5\% confidence level with both sub-samples \(F(2, 259) = 0.327\) with \(p\) value = 0.722 for the unemployment rate sub-sample and \(F(2, 259) = 0.960\) with \(p\) value = 0.385 for the real output sub-sample).

With the multivariate MRA of the unemployment sub-sample, the null hypothesis of linear functional form (no omitted variables) for the estimated model is not rejected by the Ramsey RESET test. Empirical estimates of the magnitude of the OLC obtained with models specified with unemployment rate as the endogenous variable are affected by the frequencies of the data bases (FREQSQ: +), the development level of the countries (COUNTDING: +) and the level or first difference specification of the model (LEVEL: -). The higher the frequency of the data, the smaller the OLC (in absolute terms). Whereas it may be rather rapid in some circumstance, it takes time for output variations to change the rate of unemployment. Quarterly or semestrial data bases may thus yield lower estimated OLC values. Other things equal, the estimated OLC is also lower (in absolute terms) when the data base includes only non OECD countries. This may be explained by the fact that the magnitude of the OLC depends on labour market institutions, the ease of hiring and firing workers, labour mobility, migration possibilities, and the nature of economic shocks. Finally, specification of the Okun’s Law model in levels systematically leads to higher estimated OLC values (in absolute terms). Contrary to first difference models which capture the impact or delayed effects of specific output shocks, models specified with levels of the variables may rather capture the total cumulated effect of real output variations on unemployment. The corresponding estimates of the OLC may thus be expected to be larger with this kind of model.
With the multivariate MRA of the output sub-sample, the overall fit is quite high for a meta regression and the null hypothesis of linear functional form is again non-rejected by the RESET test. The last two column of Table 5 shows that empirical estimates of the OLC obtained with models specified with some measure of output as the endogenous variable are smaller (in absolute values) when using semestrial or quarterly data instead of annual data (FREQSQ: +), when using regional data instead of national data (REG: +), with a dynamic model of the Okun’s Law involving lags of the measure of unemployment and/or real output (MODDYN: +), and when the model includes level of the variables (LEVEL: +). Moreover, a more recent data base also seem to lead to smaller values (in absolute values) of the OLC (AVGYEAR : +). In contrast, the estimated impact on unemployment of output is larger (in absolute terms) when extra exogenous variables are added to the regression model (OTHEXO: -).

These results suggest the following. First, studies that use regional data instead of macroeconomic data are more likely to report smaller values (in absolute terms) of the OLC. This lower sensitivity of unemployment rate to regional output variations may be due to the fact that asymmetric regional output shocks are partly dampened by local or regional policy adjustments. Another possibility might be that regional labour market disequilibrium is partly cancelled by real wage variations and labour mobility so that the regulation doesn’t systematically occur through variations in the number of unemployed persons. Secondly, the absolute value of the OLC tends to be smaller (in absolute terms) in studies using a dynamic model instead of a static one. Dynamic models incorporate lags of the endogenous variable and may also include lags of the exogenous variables as in the traditional auto regressive distributed lag (ARDL) model. Even with a limited number of lags, this kind of model may capture the total cumulated effect of real output variations on unemployment. This total cumulated effect of real output on unemployment may thus be expected to be lower than the impact effect evaluated with a static model if disequilibria of the labour market tend to vanish progressively over time. However, this interpretation has to be advanced with care because the retained sample does not allow us to investigate the context of complex dynamic effects such as threshold effects or nonlinear effects over time.
CONCLUSION

Empirical literature on the Okun’s Law is heterogeneous to a large extent. This paper seeks to identify whether or not there is a representative or true empirical Okun’s Law coefficient and to measure its size. If such a true effect exists, it may be considered as the representative effect of real GDP movements on variations in the unemployment rate.

We select a sample of 269 estimates of the Okun’s Law coefficient from the literature to uncover the reasons for the differences in empirical results across studies and to estimate the ‘true’ OLC. We then carry out separate MRA on two separate subsamples: the group of studies using some measure of unemployment as the dependent variable and the group of studies involving a production function version of the Okun’s Law with some measure of output as the dependent variable. While there is evidence of type II bias in both sub-samples, a type I bias is present only among the papers using some measure of unemployment as the dependent variable. Moreover, taking into account those biases, the estimated true OLCs are significantly larger (in absolute value) with models using some measure of output as the dependent variable: (-0.61 instead of -0.25 with a bivariate MRA and -1.02 instead of -0.40 with a multivariate MRA). While several recent papers show that the OLC may vary across the business cycles, across countries and across time periods, our results clearly show that one of the primary source of heterogeneity that can be identified in this literature is between studies which investigate the Okun’s Law coefficient with a model including some measure of unemployment as the dependent variable and those that focus on a model involving some measure of output as the endogenous variable. Thus, model specification is an important source of heterogeneity in this literature, and it may be reasonable to argue that there are two true values for the OLC depending of the form of the estimation model. Selecting some measure of output as the endogenous might just amount to estimating some form of a production function indicating the long-run impact of employment on real output. In contrast, when estimating the OLC with a model specified so that some measure of the unemployment rate is the endogenous variable, the model seems adequate to capture the short run impact of aggregate demand movements on unemployment variations.
Among the remaining sources of heterogeneity, the dynamics specification of the model, the frequency of the data, the degree of development of the countries and the choice between regional data and national data are particularly important.
References


