HOW MUCH DOES A SINGLE GRADUATION COHORT FROM FURTHER EDUCATION COLLEGES CONTRIBUTE TO AN OPEN REGIONAL ECONOMY?

BY KRISTINN HERMANNSSON, PATRIZIO LECCA AND KIM SWALES

UNIVERSITY OF STRATHCLYDE
How much does a single graduation cohort from further education colleges contribute to an open regional economy?

Kristinn Hermannsson*
Patrizio Lecca†
J Kim Swales‡

*School of Education, University of Glasgow, St Andrews Building, 11 Eldon Street, Glasgow G3 6NH, UK. Tel: +44 (0) 141 330 2210.
Kristinn.hermannsson@glasgow.ac.uk

†Fraser of Allander Institute, Department of Economics, University of Strathclyde. Sir William Duncan Building, 130 Rottenrow, Glasgow G4 0GE, UK. Tel: +44 (0) 141 548 3958. Patrizio.lecca@strath.ac.uk; j.k.swales@strath.ac.uk

Abstract
Econometric analysis has been inconclusive in determining the contribution that increased skills have on macroeconomic performance whilst conventional growth accounting approaches to the same problem rest on restrictive assumptions. We propose an alternative micro-to-macro method which combines elements of growth accounting and numerical general equilibrium modelling. The usefulness of this approach for applied education policy analysis is demonstrated by evaluating the macroeconomic impact on the Scottish economy of a single graduation cohort from further education colleges. We find the macroeconomic impact to be significant. From a policy point of view this supports a revival of interest in the conventional teaching role of education institutions.

Keywords: Graduates, Further Education Colleges, Labour Supply, Economic Impact, General Equilibrium

JEL Codes: I23, E17, D58, R13.

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1. Introduction

It is not clear how increasing the skills of the population through formal education affects aggregate economic activity. Neoclassical growth theory and microeconometric analysis of wage premia in labour markets motivate a prima facia expectation that more education should boost output. However, verifying this has proven elusive – macroeconometric analysis has been inconclusive, whilst conventional growth accounting approaches to the same problem rest on restrictive assumptions. Perhaps not surprisingly, given this academic context, the benefits associated with educational institutions are increasingly linked to peripheral features of their mission, such as their expenditure impacts and their knowledge exchange activities (e.g. Scottish Government, 2013), rather than their more fundamental economic contribution of providing skills.

We propose an alternative micro-to-macro method which combines elements of growth accounting and numerical general equilibrium modelling. This enables us to relax the assumption of fixed marginal products inherent in growth accounting, while retaining the flexibility obtained from building on labour market data. We demonstrate the usefulness of this approach for applied education policy analysis by evaluating the macroeconomic impact on the Scottish economy of a single graduation cohort from Scottish further education colleges (FECs). We find the macroeconomic impact to be significant and greater than would be predicted using the conventional growth accounting framework. From a policy point of view this supports a revival of interest in the conventional teaching role of education institutions. Further, a similar comparative analysis for higher education institutions (HEIs) in Scotland reveals FECs to be relatively cost effective in improving the local level of human capital.

Section 2 examines the evidence from the literature on the impact of education on aggregate economic activity and outlines our own micro-to-macro method. Section 3 calculates the increase in human capital generated in Scottish HEIs and FECs from their operation in one single year and the subsequent effect on Scottish labour productivity. Section 4 presents a stripped-down analytical model of the effect of such an increase in labour productivity in an open regional economy. These analytical results are compared with those from the standard growth accounting approach. Section 5 reports simulation results from a much more sophisticated and numerically based Computable General
Equilibrium (CGE) model for Scotland. These simulations give the impacts on economic activity of the increase in labour productivity generated by a single year’s output from Scottish FECs. Section 6 is a short conclusion.

2. Previous research into the macroeconomic impact of education

An extensive international micro-econometric literature documents the private rates of return to additional education in the form of higher earnings. Sometimes the results are further disaggregated by characteristics such as gender, social background and academic discipline. These studies reveal a clear correlation between an individual’s education level and his or her wage (see Psacharopoulous and Patrinos, 2004, for a survey). Historical observations show wage premia remaining significant and positive over extended time periods, despite increased levels of educational attainment and hence an increased supply of skilled labour. This outcome is typically attributed to skill biased technical change, which has simultaneously increased the demand for skilled labour to match the increase in skilled labour supply (Goldin & Katz, 2007; Machin, 2004). This explanation is consistent with evidence from recent work showing that despite the persistence of stable average wage premia, the variation in individuals’ wage premia has increased over time (Green & Zhu, 2010; Walker & Zhu, 2008).

Whilst there is a strong correlation between education and income, identifying causality is difficult due to an inability to conduct appropriate controlled experiments. Interpreted in the spirit of the human capital school, education directly increases human capital, which in turn increases worker productivity (Becker, 1964; Mincer, 1958; Schultz, 1960). An alternative view is motivated by the theory of signalling and screening (Arrow, 1973; Spence, 1973; Stiglitz, 1975). This approach maintains that in extremis education does not enhance human capital, and as a consequence improve productivity, but simply reveals innate ability to employers. Brown & Sessions (2004) provide an overview to the debate. A range of statistical approaches have been applied to address this conundrum, such as utilising natural experiments (Card, 2001; Krueger & Lindahl, 2001) and controlling for fixed effects using twin samples (Bonjour et al, 2003; McMahon, 2009, Appendix A). The current consensus is that education affects income per se and is not simply a proxy for unobserved ability (Blundell et al 2005; Card, 1999, 2001; Harmon & Walker, 2003). There is likely to be
some role for signalling, but of modest magnitude relative to overall impacts (Lange & Topel, 2006). These issues are discussed in detail in Hermannsson et al (2014).

Existing evidence therefore suggests that education provides a benefit at the micro level by increasing labour productivity. However this raises the question of how such a stimulus to workers’ productive capacity will impact the wider macro-economy. There is a strong prima facie argument that improved levels of education should be associated with improved national economic performance. However, this aggregate impact has proved difficult to quantify.

The most straightforward approach involves cross-country empirical work that estimates the link between education and the macroeconomy, which is reviewed in Sianesi & van Reenen (2003) and Stephens & Weale (2004). As Sianesi & van Reenen (2003) state, these studies exploit cross-country variation in the data to estimate the parameters for an aggregate production function or growth equation. Usually the regressions are based on aggregate human capital indicators as proxied by the average level of educational attainment. These human capital measures can be disaggregated into different types of education, though the extent of this disaggregation is generally limited by data availability and statistical power.

A weakness of the macro-econometric studies for informing policy debates is the wide range of results that they produce. At one extreme, some authors, such as Benhabib & Spiegel (1994) and Barro & Sala-i-Martin (1995) have been highly sceptical of the macroeconomic impact of education, though the results of their work have been contested based on the quality of the datasets used (Krueger & Lindahl, 2001). At the other extreme, approaches using endogenous growth models have suggested implausibly large impacts from education which are critically discussed in Topel (1999). Sianesi & Van Reenen (2003) survey over 20 macro growth regressions and argue that overall these provide valuable qualitative evidence on the link between education and economic output. However, in light of methodological complications they urge caution in using results to quantify the magnitude of such links.

However, even if formal education only operates as a signal it will increase the productivity of the overall economy if it improving job matching in the labour market. This occurs if the signal allows workers to be allocated to jobs that better match their ability level.
An alternative “micro-to-macro” approach to estimating the aggregate impact of education is through growth accounting (Barro, 1999; Stephens & Weale, 2004). This method simply counts and aggregates inputs in a production function, assigning marginal productivity based on available evidence. Typically the assumption that inputs are paid their marginal products is made. In such an exercise the growth in output is decomposed into the contribution made by changes in all inputs and a residual productivity growth element. The genesis of this literature is usually attributed to Solow (1956), though for a discussion of precedents, see Griliches (1996). Subsequent refinements incorporated more elaborate treatment of inputs (capital, human capital and natural resources) and emphasised more accurate estimation of their contribution, thereby gradually reducing the “residual” share of growth that could not be attributed to inputs (e.g. Jorgenson & Griliches 1967).

The strength of this approach is its transparency, and through its use a link can be made from education policy levers to the macro-economy given a suitably detailed treatment of human capital inputs. However, the method is restricted in that it relies on key simplifying assumptions. In particular, marginal products of individual inputs are taken to be fixed and given by base year input payments so that no allowance is made for endogenous adjustments in the marginal products that occur as the relative supply of inputs change. Second, all changes in inputs are taken to be exogenous. This implies that growth accounting fails to identify any subsequent changes in the supply of other inputs, such as labour and physical capital, driven by the exogenous changes in human capital. These changes in the supply of other inputs are in response to endogenous changes in their marginal products.

We adopt an alternative ‘micro-to-macro’ approach which retains the strengths of aggregating human capital inputs using micro level data, but then applies an extensive structural model to simulate subsequent endogenous adjustments in the economy (Hermannsson et al, 2014; Giesecke & Madden, 2006). This identifies the link between the policy lever and the macroeconomic outcome. Furthermore, it is possible (in principle at least) to isolate particular effects. This Computable General Equilibrium (CGE) approach has clear underpinning in neoclassical economics. It can be extended to allow for a range of endogenous adjustments and can, in principle at least, accommodate a range of views on the transmission mechanism. However, the endogenous elements are determined by the nature of the model and are sensitive to the chosen parameters.
3. The 2011 FEC graduation cohort and its productivity impact

Following Hermannsson et al (2014) we use the evidence of the comparative constancy of the graduate wage premium in recent UK history to motivate an important simplifying assumption: that we treat human capital as homogenous. Given the wide range of qualifications that the FECs provide, there seems no reasonable alternative to this strategy. The stock of human capital is then calculated following the standard growth accounting procedure, where the supply of labour at different skill levels is aggregated into a single stock of human capital, constructed as efficiency units of labour. In this aggregation, relative wage rates are taken to reflect directly skill differentials. Following Acemoglu & Autor (2012), for unskilled, $U$, and k types of skilled labour, $S_i$, the human capital stock in efficiency units, $N_E$, can be calculated as:

\[ N_E = U + \sum_{i=1}^{k} \frac{w_i}{w_U} S_i = U + \sum_{i=1}^{k} m_i S_i \]

where $w_U$ is the wage of unskilled workers, $w_i$ is the wage of skill group i and $m_i (= w_i / w_U)$ is the wage premium for skill group i.

We wish to measure the human capital that FECs generated in one academic year, $\Delta N_{E,t}$. The year is denoted by the subscript t and the FEC source by the superscript F. This human capital is calculated as the sum of the number of qualifications at each level received by FEC students in that year, $S_{i,t}^F$, weighted by the human capital gain that the recipient gets through reaching a particular qualification, $\Delta m_i$. This is shown in equations (2) and (3). We make the joint assumptions that qualifications can be represented hierarchically and that achieving an educational qualifications raises the recipient one step on that hierarchy. The additional human capital generated when an individual achieves a particular qualification is then the difference between the human capital for that qualification level and the human capital associated with the preceding qualification level.

\[ \Delta N_{E,t}^F = \sum_{i=1}^{k} \Delta m_i S_{i,t}^F \]

where
In order to use equations (1), (2) and (3) to determine the productivity impact of the 2011 cohort of graduates from Scottish FECs, we need the number of graduates who attained that qualification level in the academic year 2010/11, \( S_{i,2011} \), and the corresponding efficiency gain associated with achieving each of these qualifications, \( \Delta m_i \).

The data on the number and breakdown of qualifications were collected from the individual Scottish FECs. They were classified using the National Vocational Qualification, NVQ, scale which identifies 5 broad levels. Although these standard classifications were developed for vocational qualifications, labour market researchers have established conventions that map them to the equivalent ranking of academic qualifications. This mapping is shown in Table 1. A detailed discussion of these classifications is given in Walker and Zhu (2007a, pp. 19-21).

### Table 1: NVQ levels with the corresponding academic and vocational qualifications.

<table>
<thead>
<tr>
<th>NVQ level</th>
<th>Academic qualification</th>
<th>Vocations qualification</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>&lt;5 GCSE, General Standard Grade</td>
<td>BTEC, SCOTVEC first or general certificate</td>
</tr>
<tr>
<td>2</td>
<td>5+ GCSEs at A-C, 'O' Grades, Credit Standard Grade</td>
<td>GSVQ/NVQ intermediate, RSA diploma</td>
</tr>
<tr>
<td>3</td>
<td>2+ A-levels/3+Highers</td>
<td>OND, ONC</td>
</tr>
<tr>
<td>4</td>
<td>Undergraduate degree</td>
<td>HNC/HND</td>
</tr>
<tr>
<td>5</td>
<td>PhD, Masters degrees</td>
<td>PGCE, Non-masters postgraduate qualifications</td>
</tr>
</tbody>
</table>

Source: Walker & Zhu (2007b, Figure 4.1, p. 21).

The numbers of students attending Scottish FECs in 2011 are given in Table 2, broken down by the new qualification, if any, that they received in that year. The final column in Table 2 shows that 76,152 FEC students completed some form of programme in that year. Of these, 7,945 received no qualification and 36,136 gained a qualification that failed to raise his or her status on the NVQ scale. Such qualifications often grant access to, or prepare students
for, more advanced courses and are treated as intermediate steps between NVQ levels. They are therefore ignored in remainder of this analysis in order to avoid double counting. The remaining 32,071 students received a qualification that represents an interval on the NVQ scale, and these are identified in the rows 3 to 7 in Table 2. The figures in these rows provide the individual \( S^{F_i} \) values required for equation (2). Note that these are separated into academic and vocational NVQs and this will be indicated by a corresponding A and V superscript in subsequent analysis. Over 85% of the qualifications achieved in Scottish FECs in this year are vocational and just over 50% are for a vocational NVQ 4 level.

In order to value the economic benefit of achieving each increment on the NVQ scale, and therefore identify the appropriate \( \Delta \eta_i \) values, we draw on micro-econometric evidence on the Scottish wage premium by qualification found in Walker and Zhu (2007a, b). As discussed in Section 2, the wage premium associated with a given level of educational qualification is taken to indicate the productivity enhancing effects of education.

Table 2 Number of students successfully completing a Scottish FEC course in 2010-11, split by academic/vocational study and aggregated to NVQ level (FTEs).

<table>
<thead>
<tr>
<th>NVQ level</th>
<th>Academic</th>
<th>Vocational</th>
<th>All students</th>
</tr>
</thead>
<tbody>
<tr>
<td>No qualification</td>
<td>7,945</td>
<td>7,945</td>
<td></td>
</tr>
<tr>
<td>Intermediate NVQ</td>
<td>13,072</td>
<td>23,064</td>
<td>36,136</td>
</tr>
<tr>
<td>1</td>
<td>167</td>
<td>886</td>
<td>1,053</td>
</tr>
<tr>
<td>2</td>
<td>2,551</td>
<td>3,854</td>
<td>6,406</td>
</tr>
<tr>
<td>3</td>
<td>1,498</td>
<td>5,768</td>
<td>7,266</td>
</tr>
<tr>
<td>4</td>
<td>484</td>
<td>16,829</td>
<td>17,313</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>26</td>
<td>34</td>
</tr>
<tr>
<td>Total NVQ 1-5</td>
<td>4,708</td>
<td>27,363</td>
<td>32,071</td>
</tr>
<tr>
<td>Total all qualifications</td>
<td>17,780</td>
<td>58,372</td>
<td>76,152</td>
</tr>
</tbody>
</table>

Source: David Hume Institute.

The Scottish wage premia are shown in Table 3. Walker & Zhu (2007a, b) pool ten years of data from the Labour Force Surveys for 1996-2005. This allows them to construct a large
enough sample to estimate wage premia by academic and vocational NVQ qualification at a regional level within Great Britain. Their broad findings are in line with other work in the field; qualifications increase the probability of being employed and more qualified workers generally earn higher wages. For both men and women the impact of qualifications on wage premia is broadly similar in Scotland and Great Britain as a whole.

As is evident from the results reported in Table 3, Walker & Zhu (2007a, b) find strong wage effects for both vocational and academic qualifications in the Scottish labour market. Overall the academic qualifications yield a higher wage premia. But what is also noteworthy is how the structure of the wage premia by levels of qualification differs between vocational and academic qualifications. The marginal effect of low level vocational qualifications is modest vis-á-vis low level academic qualifications. However, the additional wage premia gained by postgraduate academic study is also relatively small. From a human capital perspective, these findings are not surprising if the amount of schooling behind these education levels is examined. For example, in Scotland a Level 4 undergraduate degree typically takes four years to complete whereas the common duration for masters’ degrees is 12 months. This implies that the wage premia earned per effective duration of study (and therefore also the return to education) should be broadly similar between Level 4 and Level 5. Walker & Zhu (2007a, b) report their results separately for each gender. For our analysis we use a simple average of the two, and therefore implicitly adopt the assumption that the gender balance is equal within each increment of the NVQ scale.

Table 3 Male, female and average hourly wage premia and average efficiency levels and efficiency gains for vocational (V) and academic (A) qualifications in Scotland.

<table>
<thead>
<tr>
<th>NVQ level</th>
<th>Vocational</th>
<th>Wage premia (%)</th>
<th>Efficiency level</th>
<th>Efficiency gain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Male</td>
<td>Female</td>
<td>Average</td>
<td>$m_i^V$</td>
</tr>
<tr>
<td>Level 1</td>
<td>9%</td>
<td>11%</td>
<td>10%</td>
<td>1.10</td>
</tr>
<tr>
<td>Level 2</td>
<td>16%</td>
<td>20%</td>
<td>18%</td>
<td>1.18</td>
</tr>
<tr>
<td>Level 3</td>
<td>35%</td>
<td>29%</td>
<td>32%</td>
<td>1.32</td>
</tr>
<tr>
<td>Level 4</td>
<td>52%</td>
<td>52%</td>
<td>52%</td>
<td>1.52</td>
</tr>
<tr>
<td>Above level 4</td>
<td>82%</td>
<td>81%</td>
<td>82%</td>
<td>1.82</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NVQ level</th>
<th>Academic</th>
<th>Wage premia (%)</th>
<th>Efficiency level</th>
<th>Efficiency gain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Male</td>
<td>Female</td>
<td>Average</td>
<td>$m_i^A$</td>
</tr>
<tr>
<td>Level 1</td>
<td>17%</td>
<td>18%</td>
<td>18%</td>
<td>1.18</td>
</tr>
<tr>
<td>Level 2</td>
<td>29%</td>
<td>30%</td>
<td>30%</td>
<td>1.30</td>
</tr>
<tr>
<td>Level 3</td>
<td>48%</td>
<td>43%</td>
<td>46%</td>
<td>1.46</td>
</tr>
<tr>
<td>Level 4</td>
<td>79%</td>
<td>77%</td>
<td>78%</td>
<td>1.78</td>
</tr>
<tr>
<td>Above level 4</td>
<td>91%</td>
<td>90%</td>
<td>91%</td>
<td>1.91</td>
</tr>
</tbody>
</table>

Source: Walker & Zhu (2007b, Figure 4.3, Figure 4.5, pp. 12-13), own calculations.
In this approach, workers contribute different amounts of efficiency units of labour to the production process, depending on their skill level. We set the efficiency units of those with no qualification to 1 and then use the evidence of the wage premium to inflate the efficiency units of each worker in accordance with his or her skill level. For example a worker with a level 1 vocational qualification contributes 1.10 efficiency units, someone with level 2 qualification 1.18 and so on. These figures are shown in column 5 of Table 3.

Given these figures it is possible to calculate the efficiency units that FEC graduates bring to the labour market. However, we are initially solely interested in the extent to which the graduates’ efficiency has increased as a result of the FEC course they have completed in the year 2010/11. That is, we want to focus on the additional skills provided by the particular course and not the skills already possessed by that worker (for example the skills gained at school). Therefore a student completing a level 3 academic qualification adds 0.16 efficiency units to his or her human capital. This is the difference between the efficiency units associated with a level 3 qualification and a level 2 qualification (1.46 – 1.30 = 0.16).

The efficiency gain generated by attaining each type of qualification is shown in column 6 of Table 3.

Table 4. The additional efficiency units from achieving particular NVQ qualifications and the estimated increase in the labour supply, measured in efficiency units, generated by the 2010-11 Scottish FEC cohort of graduates.

<table>
<thead>
<tr>
<th>NVQ level i</th>
<th>Academic</th>
<th>Vocational</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Efficiency gain $\Delta m_i^A$</td>
<td>Increased human capital $S_{F,A}^i,2011$</td>
</tr>
<tr>
<td>1</td>
<td>0.18</td>
<td>167</td>
</tr>
<tr>
<td>2</td>
<td>0.12</td>
<td>2,551</td>
</tr>
<tr>
<td>3</td>
<td>0.16</td>
<td>1,498</td>
</tr>
<tr>
<td>4</td>
<td>0.32</td>
<td>484</td>
</tr>
<tr>
<td>5</td>
<td>0.13</td>
<td>8</td>
</tr>
<tr>
<td>Total</td>
<td>4,708</td>
<td>733</td>
</tr>
</tbody>
</table>

Table 4 gives the changes in human capital, measured in efficiency units, produced by the skills that Scottish students gained in achieving academic and vocational NVQ qualifications in Scottish FECs in the academic year 2010/11. These are calculated using equation (2). The number for each qualification is taken from Table 2 and the corresponding efficiency gains
per qualification from Table 3. For each qualification, the increase in human capital is calculated by multiplying the number of successful students by the efficiency gain received from enhancing the individual’s qualifications. Therefore for vocational NVQ level 4, the increase in human capital is \(0.20 \times 16,829 = 3,366\). Using this metric, the 2010-11 output from Scottish FECs increased the effective Scottish labour supply by 5,311 efficiency units, primarily through the provision of vocational NVQs.

To illustrate the scale and relative importance of the FEC impact for Scotland it is useful to estimate the equivalent human capital output of Scottish Higher Education Institutions (HEIs) as a benchmark. For this comparison, we focus only on those students that are funded by the Scottish Funding Council, i.e. Scottish domiciled students and students from the European Union outside the UK. For simplicity we assume that only the Scottish students are retained in Scotland following graduation.

Information on the number of Scottish domiciled students graduating from Scottish universities are obtained from the Higher Education Statistics Agency (HESA). We use exactly the same method as for FECs. We count the output as qualifications completed in the academic year 2010/11. For HEIs this represents undergraduate, higher and doctoral qualifications. In Table 5 these figures are presented and compared to the FEC data. Scottish FECs provide a greater absolute number of qualifications over a much wider range of skills than do the HEIs. However, the Scottish HEIs contribute 46% more human capital in terms of efficiency units. This means that for 2010/11 FECs contributed just over 40% of the human capital generated in the combined Scottish HEI and FEC sectors.

### Table 5 Qualifications completed and the associated marginal increase in efficiency units at Scottish FECs and HEIs, 2010/11

<table>
<thead>
<tr>
<th>NVQ level</th>
<th>FECs</th>
<th>Efficiency units</th>
<th>HEIs</th>
<th>Efficiency units</th>
<th>Total</th>
<th>Efficiency units</th>
</tr>
</thead>
<tbody>
<tr>
<td>NVQ1</td>
<td>1,053</td>
<td>118</td>
<td>0</td>
<td>0</td>
<td>1,053</td>
<td>118</td>
</tr>
<tr>
<td>NVQ2</td>
<td>6,406</td>
<td>615</td>
<td>0</td>
<td>0</td>
<td>6,406</td>
<td>615</td>
</tr>
<tr>
<td>NVQ3</td>
<td>7,266</td>
<td>1,047</td>
<td>0</td>
<td>0</td>
<td>7,266</td>
<td>1,047</td>
</tr>
<tr>
<td>NVQ4</td>
<td>17,313</td>
<td>3,523</td>
<td>21,875</td>
<td>7,109</td>
<td>39,188</td>
<td>10,632</td>
</tr>
<tr>
<td>NVQ 5</td>
<td>34</td>
<td>9</td>
<td>5,135</td>
<td>642</td>
<td>5,169</td>
<td>651</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>32,071</td>
<td>5,311</td>
<td>27,010</td>
<td>7,751</td>
<td>59,081</td>
<td>13,063</td>
</tr>
</tbody>
</table>
From a policy perspective, it is of interest to calculate the cost to the Scottish Government, through the Scottish Funding Council, of generating human capital using these different educational institutions. This is calculated in the following way. The funding for tuition costs for Scottish students going to Scottish HEIs and FECs in the academic year 2010/11 divided by the total additional human capital, as manifested in completed NVQ increments, generated in those sets of institutions in the same year. As revealed in Table 6, whilst FECs produce just over 40% of the human capital, they receive slightly less than 40% of the funding. From the point of view of public funding, the average cost per efficiency unit of human capital is just over £101,000, with the cost per unit around 8% higher in Scottish HEIs (£105,000) than in Scottish FECs (£97,000).³

### Table 6 Scottish Funding Council cost per additional efficiency unit at FECs and HEIs, 2010/11

<table>
<thead>
<tr>
<th></th>
<th>FECs</th>
<th>HEIs</th>
<th>Total / average</th>
</tr>
</thead>
<tbody>
<tr>
<td>SFC contribution (less research)</td>
<td>515,404</td>
<td>812,156</td>
<td>1,327,560</td>
</tr>
<tr>
<td>Marginal increase in efficiency units</td>
<td>5,311</td>
<td>7,751</td>
<td>13,063</td>
</tr>
<tr>
<td>Cost per efficiency unit £</td>
<td>97,040</td>
<td>104,777</td>
<td>101,631</td>
</tr>
</tbody>
</table>

Whilst it is of interest to compare the relative scale and cost effectiveness of the Scottish FEC and HEI sectors, the main concern of this paper is to measure the aggregate economic impact of Scottish FECs. In particular we wish to simulate the impact of the increase in labour efficiency generated by the human capital from one year’s output of achieved qualifications. This requires that we first calculate the proportionate change in Scottish total human capital that one year’s FEC output would produce.

Drawing on the Annual Population Survey (APS), it is possible to obtain the number of those between the age of 16 and 64 in Scotland together with their skill level. This is based on several simplifying assumptions. The APS is accessed via the National Online Manpower Information System (NOMIS) data portal of the Office for National Statistics (ONS). This data

³ It is important to note that this is not a full cost effectiveness calculation. It only covers the opportunity cost to Scottish citizens in terms of the foregone public expenditure on other goods and services that could have been otherwise provided or subsidised by the Scottish Government. Furthermore, it only includes the funding of the education institutions themselves and excludes other student-related expenditures, such as maintenance grants.
source aggregates NVQ4 and NVQ5 qualifications to avoid disclosure. Therefore we abstract from the role of NVQ5 qualifications in the skills base. That is to say, if individuals are recorded as having a skill level of SVQ 4+ we allocate them the efficiency level appropriate for NVQ 4. Furthermore, the APS does not distinguish between academic and vocational qualifications. Therefore we use the average wage premium for a given NVQ level, which implies that within each skill increment, academic and vocational qualifications are assumed to be in equal measure. Finally, those with 'Other qualifications' or where information is not available make up 6% and 5% of the population respectively. These respondents are treated as though they had no qualification.

Using the wage premia as reported in Walker and Zhu (2007) we calculate the efficiency units of labour contained in each individual and sum these across the whole working-age population. According to the APS there were 3,378,700 individuals aged 16-64 in Scotland in 2011. This population could supply 4,560,838 efficiency units of labour, which suggest that the average number of efficiency units of labour per working age Scot is 1.35. Using this figure as a denominator we find that our 2011 graduation cohort has increased the amount of available efficiency units of labour by 0.12% (5,311 / 4,560,838 = 0.0012).

4. Analytical model

In this section we present a stripped-down analytical model of an open regional economy subject to a labour efficiency shock, such as would occur with an increase in human capital. In particular we want to compare the results generated by such a model with those associated with the standard growth accounting approach. The analytical model presented in this section is expressed in terms of proportional changes and is focused on the long-run impact on aggregate output. All domestically produced goods are sold in an export market and all consumption and investment goods are imported. As we assume that there is no change in import prices, this allows us to abstract from the distinction between changes in real and nominal wages. Production is undertaken with no intermediate inputs, so that output is identical to value added (and GDP). The labour force is fixed but employment can vary through changes in the employment rate. Investment and capital stock are endogenous with the cost of capital fixed and determined by interest rates set in extra-regional markets. There is no government expenditure or taxes.
An increase in labour efficiency, \( \rho (>0) \), increases the labour change measured in efficiency units, \( n_E \), supplied by a given labour change measured in natural units, \( n \). This can be expressed as:

\[
(4) \quad n_E = n + \rho
\]

Similarly, the changes in the wage for a unit of labour measured in natural and efficiency units (\( w \) and \( w_E \)) can be represented as:

\[
(5) \quad w_E = w - \rho
\]

The change in the real wage, \( w \), is positively related to the change in employment through a bargaining function, all variables here measured in natural units:

\[
(6) \quad w = \beta n \quad \beta \geq 0
\]

where \( \beta \) is the elasticity of the real wage with respect to the employment rate. This is the familiar wage curve relationship (Blanchflower and Oswald, 1994; 2005). The change in product demand, \( q \), is negatively related to the change in product price, \( p \), so that:

\[
(7) \quad q = -\eta p \quad \eta \geq 0
\]

where \( \eta \) is the price elasticity of demand. Recall that all output is sold in export markets, so that equation (7) implies that the region’s output is a less than perfect substitute for other goods in such markets. This corresponds to the familiar Armington assumption (Armington, 1969). The change in labour productivity, measured in in efficiency units, can be described as

\[
(8) \quad q - n_E = \sigma (w_E - p) \quad \sigma \geq 0
\]

where \( \sigma \) is the elasticity of substitution between capital and labour in production (Heathfield and Wibe, 1987, p. 93). There is also a corresponding equation for capital productivity.
\[ q - k = -\sigma p \]

where \( k \) is the change in capital stock and the change in the cost of capital is zero by assumption. Finally, product price change is represented as:

\[ p = \alpha w_E \quad 1 \geq \alpha > 0 \]

where \( \alpha \) is the share of labour inputs in production, given that the change in the price of capital is zero.

In this very simple model, we have seven equations to solve for the seven endogenous variables: \( k, n, n_E, w, w_E, p \) and \( q \). Their values are driven by the exogenous change in the efficiency of labour, \( \rho \), and the elasticity values \( \beta, \eta \) and \( \sigma \), together with the labour share parameter \( \alpha \). To begin, we can solve for the proportionate changes in the use of the two factors of production, labour and capital, \( n \) and \( k \). These relationships, which are derived in Appendix 1, are given as:

\[
(11) \quad n = \frac{\rho(\sigma(1-\alpha) + \eta \alpha - 1)}{\beta(\sigma(1-\alpha) + \eta \alpha) + 1}
\]

and

\[
(12) \quad k = \alpha \rho \frac{(1 + \beta)(\eta - \sigma)}{\beta(\sigma(1-\alpha) + \eta \alpha) + 1}
\]

Equations (11) and (12) indicate that the increase in labour efficiency generates endogenous changes in employment and capital use that can be positive or negative. Given the constraints on the parameter values, the denominator in equations (11) and (12) is always positive. Therefore the sign of the effect on the factor use depends on the sign of the numerator in the corresponding equations.

Employment change is positive so long as \( \eta \alpha + \sigma(1-\alpha) > 1 \). That is to say, as long as the weighted sum of the product demand and elasticity of substitution elasticities is greater.
than unity. This is the requirement for the stimulus to labour demand coming through the expansion in output and the substitution of labour for capital in production to be greater than the negative impact of the increased productivity of labour in producing output. The increase in output is driven by a reduction in product price (increased competitiveness): the positive substitution effect is generated by the fall in the price of labour measured in efficiency units. For capital use the requirement is more straightforward. Capital use increases as long as the stimulus to capital demand from the output effect is greater than the negative impact of the substitution effect. This occurs where the price elasticity of demand for output is greater than the elasticity of substitution in production: that is, where \( \eta > \sigma \).

The expression for the change in output, \( q \), again derived in Appendix 1, is:

\[
(13) \quad q = \alpha \rho \frac{\eta(\beta + 1)}{\beta(\sigma(1 - \alpha) + \eta \alpha) + 1} \geq 0
\]

Given the restrictions imposed on parameter values, equation (13) shows that output always moves in the same direction as the change in efficiency. Therefore when labour efficiency increases, output will not fall. However, it is more useful to benchmark the result shown in equation (13) against the standard growth accounting figure, \( g \), for output growth. This is calculated as the proportionate increase in labour efficiency weighted by the share of labour in the production of output:

\[
(14) \quad g = \alpha \rho
\]

Recall that under growth accounting, all factor inputs are assumed to be exogenous and all output can be sold at the existing price. In contrast, in the more extensive, though still extremely simple, model represented by equations (4) – (10) factor supplies are endogenous and any change in output typically generates changes in the relative prices of output and the two productive inputs. These changes in relative prices drive, and are driven by, the endogenous changes in factor supplies. These are the changes identified in equations (11) and (12).
Using equations (13) and (14), the ratio, $R$, between the output change given by the model and by growth accounting is derived as:

\[ R = \frac{q}{g} = \frac{\eta(\beta + 1)}{\beta(\sigma(1-\alpha) + \eta \alpha) + 1} \geq 0 \quad \frac{\partial R}{\partial \eta} > 0, \frac{\partial R}{\partial \sigma} < 0 \]

This ratio depends on the values of all the exogenous parameters. It is always non-negative, which means that the two measures, $g$ and $q$, cannot move in opposite directions. Further, whilst we sign the partial differential of $R$ with respect to $\eta$ and $\sigma$, we cannot for $\alpha$ and $\beta$.

However, a key result is that $R$ can take a value greater or less than one. This implies that the standard growth accounting figure could either over- or under-estimate the impact of increased labour efficiency given by the stylised model represented by equations (4) to (10).

The most straightforward way to investigate this is to set $R \geq 1$ in equation (15) and then solve for the value of one of the parameters. The most tractable case involves solving for the elasticity of export demand, $\eta$. Following this procedure, for $R$ to be greater than 1, so that the growth accounting underestimates the impact of the labour efficiency change:

\[ \eta \geq \frac{\beta \sigma(1-\alpha) + 1}{\beta(1-\alpha) + 1} \]

This means that if the production function is Cobb-Douglas ($\sigma=1$), then growth accounting will underestimate the impact of an increase in efficiency if the demand for the product is price elastic ($\eta>1$). Further, expressing equation (16) as an equality:

\[ \frac{\partial \eta}{\partial \sigma} = \frac{\beta(1-\alpha)}{\beta(1-\alpha) + 1} < 1. \]

This implies that if the elasticity of substitution in production is less than unity, then growth accounting will underestimate the impact of an increase in labour efficiency at even lower levels of the price elasticity of demand.

Repeating this procedure for the other parameters gives less straightforward expressions, which are reported in Appendix 2. It is clear that the micro-to-macro growth accounting procedure can under- or over-estimate the impact of productivity improvements brought
about by an increase in human capital and that even in the stylised model developed in this section, the impact of labour efficiency changes on output is primarily an empirical matter. It depends upon key parameter values underlying important structural and behavioural relationships within the economy. We therefore pursue the analysis further through simulation using a computable general equilibrium model. This also allows an increase in the detail and scope of the investigation through greater industrial disaggregation, the incorporation of intermediate inputs, the recognition of domestic production for consumption, the operation of the public sector and the generation of period-by-period results.

5. CGE model results

To simulate the system-wide impact of this increase in human capital we employ AMOS, a computable general equilibrium (CGE) model of Scotland. AMOS is an acronym for A Macro-micro Model Of Scotland. This is a much extended version of the simple analytical model presented in Section 4. It is a fully specified, empirical implementation of a regional, inter-temporal, general equilibrium variant of the Layard, Nickell and Jackman (1991, 2005) model. It has: three domestic transactor groups, namely households, corporations and government; four major components of final demand: consumption, investment, government expenditure and exports; and 25 industrial sectors.

In the version of the model used in this paper, consumption and investment decisions reflect inter-temporal optimization with perfect foresight (Lecca et al, 2013, 2014). In the period-by-period simulations each period is taken to be a year. This is the period adopted in the econometric work used to parameterise the wage, migration and investment equations. Real government expenditure is exogenous. The demand for Scottish exports to the Rest of the UK (RUK) and Rest of the World (ROW) is determined via conventional export demand functions where the price elasticity of demand is set at 2.0. Imports are obtained through an Armington link (Armington, 1969) and therefore relative-price sensitive with trade substitution elasticities of 2.0 (Gibson, 1990). We do not explicitly model financial flows, our assumption being that Scotland is a price-taker in financial markets.

It is assumed that production takes place in perfectly competitive industries using multi-level production functions. This means that in every time period all commodity markets are in equilibrium with price equal to the marginal cost of production. Value-added is produced
using capital and labour via standard production function formulations so that, in general, factor substitution occurs in response to changes in relative factor-prices. Constant elasticity of substitution (CES) technology is adopted in the production of value added with elasticities of substitution of 0.3 (Harris, 1989). In each industry intermediate purchases are modelled as the demand for a composite commodity with fixed (Leontief) coefficients. These are substitutable for imported commodities via an Armington link, which is sensitive to relative prices. The composite input then combines with value-added (capital and labour) in the production of each sector’s gross output. Cost minimisation drives the industry cost functions and the factor demand functions.

In the simulations reported in this paper, the labour market is characterised by a regional bargaining function in which the bargained real wage is inversely related to the unemployment rate. The bargaining function is parameterised using the regional econometric work reported in Layard et al (1991, 2005). Population is taken to be fixed implying that the inter-regional migration function is turned off. Detailed discussion of the AMOS model and the underlying structural equations are available in Harrigan et al (1991) for the basic variant and in Lecca et al (2013, 2014) for the inter-temporal extensions. The model is calibrated to a Scottish Social Accounting Matrix (SAM) for 2006. This calibration process implies that the economy is taken to be initially in long-run steady-state equilibrium. This means that if there are no changes in the exogenous variables in the model, the simulated economy would simply reproduce the base values for every period.

As reported in Section 3, the direct impact of the 2011 cohort of graduates from FECs in Scotland is to increase labour productivity by 0.12%. It is assumed that the productivity improvement associated with this one cohort of FEC students operates over the 40 years whilst these students are assumed to remain in the labour force. To simulate the impact of such an economic disturbance we actually run the model for 80 periods, where each period represents one year. We introduce a 0.12% step increase to labour efficiency in all sectors of the economy in period 1 and maintain this for 40 periods. The stimulus is then removed for the remaining 40 periods of the simulation. The increase in labour efficiency is the only exogenous change introduced into the model, so that the results should be interpreted as deviations from what would have occurred if labour productivity had remained unchanged. The simulation identifies the supply side impact of one year’s output of Scottish FECs.
Figure 1 shows the evolutions of the GDP and employment impacts. These figures are reported as percentage changes measured against their base-year levels. Whilst the simulations are run for 80 periods, for pedagogic reasons we only report the first 60 periods as the economy has essentially returned to its initial equilibrium by that point. As can be seen, the economy reaches a plateau of higher output and employment quite rapidly, reflecting the forward-looking behavioural assumptions of the model. The maximum GDP increase of 0.126% is reached in period 14 and is retained until period 32. However, by period 4, the increase in GDP, at 0.104%, has reached 80% of its maximum value. Employment change in initially negative but becomes positive by period 4. It reaches its maximum level of 0.012% in period 13 and retains this until period 33. Once the FEC cohort leaves the labour force in period 41, there remains a short period where a legacy effect occurs, including a sharp stimulus to employment in period 41.

Table 7 Impact on key economic variables in periods 1, 30 and 40 of a temporary 0.12% increase in labour productivity lasting for 40 periods (% changes from base).

<table>
<thead>
<tr>
<th>Period</th>
<th>1</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>0.068</td>
<td>0.126</td>
<td>0.111</td>
</tr>
<tr>
<td>Consumer Price Index</td>
<td>-0.021</td>
<td>-0.059</td>
<td>-0.059</td>
</tr>
<tr>
<td>Unemployment Rate</td>
<td>0.231</td>
<td>-0.189</td>
<td>-0.036</td>
</tr>
<tr>
<td>Total Employment</td>
<td>-0.015</td>
<td>0.012</td>
<td>0.002</td>
</tr>
<tr>
<td>Nominal Wage</td>
<td>-0.048</td>
<td>-0.038</td>
<td>-0.055</td>
</tr>
<tr>
<td>Real Wage</td>
<td>-0.026</td>
<td>0.021</td>
<td>0.004</td>
</tr>
<tr>
<td>Replacement Cost of Capital</td>
<td>-0.020</td>
<td>-0.053</td>
<td>-0.053</td>
</tr>
<tr>
<td>Households Consumption</td>
<td>0.019</td>
<td>0.052</td>
<td>0.051</td>
</tr>
<tr>
<td>Investment</td>
<td>0.212</td>
<td>0.113</td>
<td>0.089</td>
</tr>
<tr>
<td>Capital Stock</td>
<td>0.000</td>
<td>0.115</td>
<td>0.089</td>
</tr>
<tr>
<td>Export to RUK</td>
<td>0.030</td>
<td>0.096</td>
<td>0.094</td>
</tr>
<tr>
<td>Export to ROW</td>
<td>0.037</td>
<td>0.088</td>
<td>0.087</td>
</tr>
</tbody>
</table>

The impacts on a wider range of key economic variables are shown for periods 1, 30 and 40 in Table 7. Again these are given as percentage changes from their base-year values. Periods 5 In order to operationalize the forward-looking model we need to impose long-run equilibrium condition in the final period (period 80). However, as is clear from Figure 1, the model is effectively in long-run equilibrium by period 60. 6 The relatively lower effects on economic activity in the short run are driven by the assumption that we introduce an unannounced efficiency shock into an economy taken to be initially in long-run equilibrium. Therefore the economy takes some time to adjust to this unanticipated supply-side shock. The medium-term impacts are a better measure of the continuing impact of one year’s output from a stable FE system.
1 is the short run, where the capital stock is fixed both in aggregate and across industrial sectors. Period 30 represents a year in which the labour efficiency impacts have reached a stable, maximum level and period 40 is just prior to the efficiency improvement’s being withdrawn.

The labour efficiency shock is modelled as if every employee can subsequently produce 0.12% more output (other things being equal). The corollary is that with the existing choice of production techniques, 0.12% less employment is required for every unit of output. This produces a short-run (period 1) increase in GDP of 0.068% together with downward pressure on prices. Exports to both the rest of the UK (RUK) and the rest of the world (ROW) rise but this is accompanied by a fall in employment of 0.015%. However, these changes trigger further adjustments in the economy. Increased labour productivity stimulates the return to capital which, in turn, leads to an increase in investment. This expansion in capacity produces further price reductions and a subsequent additional stimulus to exports. Subsequently a positive employment impacts also produce a further stimulus to GDP through increases in household consumption. The results for period 30 therefore show a rise in GDP, capital stock and employment of 0.126%, 0.115% and 0.012% respectively. The competitiveness of the economy has improved with a larger fall in the cpi and exports to the RUK and ROW are now higher by 0.096% and 0.088%. The unemployment rate has fallen, with a subsequent rise in the real wage of 0.021%.

By the end of the first 40 periods, the increases in GDP, 0.111%, and employment, 0.002%, are lower than at their peak. A comparison of the results for period 40 with those for period 30 reveals that the export and household consumption figures are very similar in the two periods. However, in period 40 there is a markedly lower value for the increase in capital stock, at 0.089%, and a particularly low increase in investment at 0.008%. This reflects the behaviour of forward looking agents adjusting to the future removal of the productivity stimulus. However, the fact that the capital stock in period 40 is greater than in the base year generates a continuing supply-side benefit to the economy in terms of increased competitiveness after period 40. This is reflected in higher GDP and employment levels. As the capital stock adjusts back to its original level this positive supply-side effect unwinds.
It is of interest to compare the change in GDP identified in Figure 1 with the change that would have been predicted using standard growth accounting methods. First, the growth accounting approach would have calculated the increase in GDP as just operating over the 40 periods during which the skill-enhanced cohort remained in the labour market. There would be no identified legacy effects. Second, the annual GDP impact would be the proportionate increase in labour productivity times the share of labour in Scottish GDP (as revealed in the Scottish Input-Output tables). This equals $0.12 \times 0.62 = 0.074$. In each of the first 40 periods after period 1, the simulation results shown in Figure 1 are greater than this figure. Once the maximum GDP change plateau is reached, that is once the full capital adjustments have been made, the simulation model generates a GDP increase 70% higher than that predicted using the growth accounting approach.

The reasons for this divergence are straightforward. As is apparent from the figures reported in the second column of results of Table 7, at its maximum the increase in labour efficiency generates an endogenous increase in employment and capital stock of 0.012% and 0.115% respectively. Their contribution to increased GDP would not be captured using
the conventional growth accounting framework. Further, this increase in capital stock continues to have a positive effect on GDP and employment after the direct efficiency increase has been withdrawn.

It is also important to recognise that the CGE model incorporates the fall in the price of human capital as a whole as its supply increases. By period 30, the real wage rises by 0.02% but this is less than the increase in human capital (0.12%). Using equation (5) we can see that the price of labour measured in efficiency units has fallen by 0.10%. This has clear distributional implications, especially for those workers who have not increased their skills and whose overall wage is squeezed as a consequence. However, in practice the role of FECs is often to provide training for those at the lower end of the skills spectrum. Therefore the skills provided by the FECs can be seen as offsetting some of the competitive disadvantage incurred by non-graduate workers as HE participation has increased. It is clear however, that those workers that are not investing in human capital are even more disadvantaged as the average skill level of the labour supply increases.

6. Conclusions

Increasingly, the benefits associated with educational institutions are those linked to peripheral features of their mission, such as their expenditure impacts and their knowledge exchange activities (Scottish Government, 2013). These benefits are undoubtedly important, especially at the local level. However, their more fundamental contribution of educational institutions is the increased skills that they generate. The benefits from this activity are often overlooked, perhaps because of the difficulty in measuring the full economic impact of such skill acquisition. This neglect is likely to have affected decisions over the allocation of public resources between physical and human capital and, within education itself, between HEIs and FECs. Specifically, because FECs’ activities are so heavily focussed on improving the skills of the local labour force, their importance might have been undervalued. Certainly in the recent economic downturn, Scottish HEIs were subject to less austerity than FECs, many of which are now in the process of undertaking radical structural change.

In this paper we have attempted to identify the supply-side effect of one year’s output from the Scottish FECs. This impact has been captured solely through the increase in human capital and the subsequent effect on GDP. We use a micro-to-macro modelling method that adopts key elements of the growth accounting method but is much less restrictive than that
approach. We are aware that there is a literature that identifies various non-monetary benefits of education to its recipient, as well as the wider monetary and non-monetary impacts of education on society as a whole (see, for example, McMahon (2004, 2009) and Hermannsson et al (2012b)). Whilst these wider impacts should be taken into account when decisions are taken on the appropriate support for education, as yet they are less well understood and analysis for FECs in Scotland along these lines would inevitably be speculative.

Even so, the estimates that we provide need to be treated with appropriate caution. They are an initial attempt to quantify this important impact. We have had to make a number of assumptions to quantify the supply-side effect of the increased efficiency associated with higher levels of human capital. However, the modelling results suggest a continuing positive impact on GDP from one year’s cohort from Scottish FECs generating a 0.126% increase in GDP over a number of decades. This estimate is significantly greater than would be predicted using standard growth accounting techniques. Moreover, the effect of FECs in improving human capital in Scotland is substantial, just over 40% of that generated in Scottish FECs and HEIs combined. Further, from our estimates Scottish FECs are cost effective in generating labour market skills; the public expenditure per unit increase in human capital is almost identical for Scottish HEIs and FECs.
Appendix 1: Derivation of the expressions for $q$ and $n$

Substituting equation (6) in the text into (5) in the text gives:

\[(A1.1) \, w_E = \beta n - \rho\]

Substituting equation (10) in the text into (8) in the text produces:

\[(A1.2) \, q - n_E = \sigma w_E (1 - \alpha)\]

Substituting equations (7) and (10) in the text into (A1.2) to eliminate $q$ and rearranging produces:

\[(A1.3) \, n_E = -w_E (\sigma (1 - \alpha) + \eta \alpha)\]

Substituting equation (5) in the text into (A1.3) and rearranging gives:

\[(A1.4) \, w_E = -\frac{n + \rho}{\sigma (1 - \alpha) + \eta \alpha}\]

Substituting (A1.4) into (A1.1) and rearranging produces equation (11) in the text determining $n$. Combining equations (5), (7) and (10) in the text gives:

\[(A1.5) \, q = -\eta \alpha (w - \rho)\]

Substituting equation (6) in the text into (A1.5) then produces:

\[(A1.6) \, q = \eta \alpha (\rho - \beta n)\]

Then substituting equation (11) in the text into (A1.7) and rearranging produces equation (13) in the text giving the value of $q$.

Rearranging equation (9) in the text and substituting in equation (7) in the text to eliminate $q$ gives:
Substituting equation (10) in the text and equation (A1.4) into (A1.7) produces:

\[ k = (\sigma - \eta)p \]  

Then substituting equation (A1.4) into equation (A1.8) gives equation (12) in the text determining $k$. 

\[ k = \alpha(\sigma - \eta)w_k \]
Appendix 2: Parameter values for which $q \geq g$

Set $R \geq 1$ in equation (15) and then solve for the parameters $\beta$, $\sigma$ and $\alpha$ produces:

\begin{align*}
(A2.1) \quad \beta & \geq \frac{1 - \eta}{\eta - (\sigma(1 - \alpha) + \alpha \eta)} \\
(A2.2) \quad \frac{\eta(1 + \beta(1 - \alpha)) - 1}{\beta(1 - \alpha)} & \geq \sigma \\
(A2.3) \quad \frac{\eta(1 + \beta) - \beta \sigma - 1}{\beta(\eta - \sigma)} & \geq \alpha
\end{align*}

It should be stressed that inequalities (A2.1), (A2.2), (A2.3) and inequality (15) in the text are simply different ways of expressing the same requirement.
References


