On the characterisation and measurement of the welfare effects of income mobility from an ex-ante perspective

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Abstract
The paper employs a rank-dependent formulation of the social welfare function with time-separable utilities to evaluate the economic consequences of income mobility from an ex-ante perspective. The resultant class of measures can be decomposed not only in terms of structural and exchange mobility but also in terms of vertical and horizontal mobility, thereby encompassing two of the main approaches in the literature. We illustrate our measurement framework by comparing mobility in the USA and Germany using data from the Cross-National Equivalent File 1980-2005. We find that the pattern of income mobility in the USA was both less pro-poor and more horizontally inequitable than in Germany, but that the latter did not translate into higher levels of exchange mobility given higher levels of absolute inequality and the vertical stance of the growth process.

Keywords: income mobility, ex-ante welfare analysis, USA, Germany.

JEL classifications: D31; D63

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1. Introduction

A number of recent developments in the measurement of income mobility have been motivated by a desire to better understand the sources of observed changes in cross-section or snapshot inequality over time. The basis for much of this work has been the distinction between ‘structural’ and ‘exchange’ mobility, originally derived from the sociology literature, where the former is identified with changes in the location and shape of the marginal income distribution and the latter with changes in the ranking of individuals within the distribution (see, for example, Silber, 1995; Ruiz-Castillo, 2004; van Kerm, 2004). A key feature of this approach is that the reranking effect is generally perceived to be a socially desirable phenomenon, consistent with the view that exchange mobility serves to equalise incomes over the longer term (see also Yitzhaki and Wodon, 2004; Fields 2005).

Mobility may also usefully be considered as a form of redistribution, which serves to emphasise the formal similarities with the analysis of the properties of tax and benefit schemes. In particular, Benabou and Ok (2001) argue that mobility is socially desirable if it is progressive in the sense that the ratio of expected income growth to initial income declines with the initial level of income. Nonetheless, it is apparent that their proposed summary index of equalizing mobility, based on Reynolds and Smolensky (1977), does not capture the whole of the redistributive impact of mobility because it fails to allow for classical horizontal inequities associated with the dispersion of individual growth paths about conditional expected rates: if individual utilities are concave in income then welfare will be higher if all incomes grow at conditional expected rates. Moreover, as is acknowledged by Benabou and Ok (2001), if individuals are risk averse then the conditional expected growth rate will be preferred to the prospect of facing a lottery yielding the same outcome on average. From either perspective, the conditional dispersion of income prospects will appear to be a socially undesirable phenomenon.

Shorrocks (1978, p.1016) argues that “interest in mobility is not only concerned with movement but also predictability – the extent to which future positions are dictated by the current place in the distribution.” The main contribution of this paper is to propose a
framework for the measurement of mobility, based on the integrative approach of Duclos et al. (2003) to the measurement of redistribution, that can account for the reranking of individuals and classical horizontal inequities as distinct phenomena, even though both will usually arise from the divergence in the fortunes of those with initially identical incomes. Within this framework, welfare gains due to income mobility are evaluated from an ex-ante, risk-neutral perspective and expressed in terms of changes in equally distributed equivalent (ede) incomes. The resultant class of measures may be decomposed either in terms of structural and exchange mobility or in terms of vertical mobility, which is determined by the scale and distribution of income growth, and horizontal mobility, which reflects the inequity associated with the conditional dispersion of individual income changes. We subsequently extend our framework to take account of risk aversion by assuming that individuals consider some certainty equivalent when evaluating the set of utility opportunities that they face. Like Gottschalk and Spolaore (2002), our analysis reveals a tension between the perceived value of reranking or reversals on the one hand and the predictability of future incomes on the other, but based on a rank-dependent formulation of the social welfare function with time-separable utilities.

The paper is organised as follows. Section 2 introduces the proposed measures to characterise and quantify mobility within a two-period framework in which the focus is on a single transition from an initial to a final state. In Section 3 we use our measures to compare mobility patterns in the USA and Germany. Section 4 concludes.

2. Measurement of mobility within a two-period framework
We follow much of the literature in focusing on a single transition between two periods and establish an ‘ethical’ basis for the measurement of mobility within this framework. This allows us to define a measure of total mobility as the sum of separate structural and exchange mobility indices. We subsequently demonstrate that this total mobility index may also be decomposed into a vertical mobility measure, which may be further split into growth and vertical redistributive components, and a horizontal mobility measure that
reflects classical horizontal inequities in the mobility process. Finally we extend the analysis to take account of the uncertainty of future income outcomes.

Let \( F(y_1, y_T) \) be the joint cumulative distribution function (cdf) for initial and final period incomes, \( Y_1 \) and \( Y_T \) respectively, with support contained in the positive real orthant. Let \( p = F_i(y_i) \) be the marginal cdf for initial period incomes, where \( p \) is the proportion of the population with an initial income less than \( y_i \). The corresponding quantile function will be \( Y_i(p) = F_i^{-1}(p) \) for \( p \in [0,1] \), which may loosely be thought of as the income of an individual with a (normalised) rank of \( p \) in the period 1 distribution. Average income in period 1 is \( \bar{y}_1 = \int_0^1 Y_1(p) \, dp \). The \( q \)-quantile function for final period incomes, \( Y_T(q) \), and average final income, \( \bar{y}_T \), are defined analogously.

We employ a social evaluation function that was first proposed by Berrebi and Silber (1981) and has more recently been used by Duclos et al. (2003) to analyse the redistributive effects of taxes and transfers in a continuous setting. This function allows us to define social value as the weighted average of individuals’ intertemporally separable utilities, where the weights are determined by individuals’ ranks in the income distribution. Thus welfare in period \( t \) evaluated on the basis of contemporaneous ranks \( r \) is equal to:

\[
W_t = \int_0^1 U(Y_t(r), \varepsilon) w(r, v) \, dr; \quad \{t, r\} = \{1, p\}, \{T, q\}
\]

(1)

where utility is given as an isoelastic function of income:

\[
U(y_t, \varepsilon) = \begin{cases} 
  y_t^{1-\varepsilon}, & \text{when } \varepsilon \neq 1 \\
  \ln(y_t), & \text{when } \varepsilon = 1
\end{cases}; \quad \varepsilon \geq 0
\]

(2)

and the rank-dependent weights \( r (r = p, q) \) are given by:

\[
w(r, v) = v(1-r)^{(v-1)}; \quad v \geq 1.
\]

(3)

Equation (1) includes the utilitarian Atkinson social welfare function (Atkinson, 1970) and the rank-dependent S-Gini class of welfare functions (Donaldson and Weymark, 1980, 1983; Yitzhaki, 1983) as special cases, when \( v = 1 \) and \( \varepsilon = 0 \) respectively. The ‘distributional judgement’ parameter \( v \) controls the rate at which the weights decrease
from poorest to richest. Specifically, $v=2$ leads to weights that decrease linearly with $p$ from 2 to 0, as with the conventional Gini coefficient. Values of $v>2$ yield indices that give greater social weight to the utility of poorer individuals than implied by the conventional Gini. Conversely, values of $v<2$ yield indices that give lesser weight to poorer individuals. In the limit $v=1$ and the social weights are independent of rank. The parameter $\varepsilon$ is the well-known measure of relative inequality aversion introduced by Atkinson (1970). Smaller values of $\varepsilon$ are associated with lower aversion to relative income inequality. The limiting case of $\varepsilon=0$ implies inequality neutrality.

Following Atkinson (1970), let the ede income of $F_t(y_t)$ be the level of income per head which if equally distributed would give the same level of social welfare:

$$
\xi_t = U^{-1}(W_t) = \begin{cases} 
\left( \int_0^1 U(Y_t(r), \varepsilon) w(r, \nu) dr \right)^{1/\varepsilon}, & \text{when } \varepsilon \neq 1 \vspace{1em} \\
\exp\left( \int_0^1 U(Y_t(r), \varepsilon) w(r, \nu) dr \right), & \text{when } \varepsilon = 1
\end{cases}
$$

(4)

where $W_t$ is as defined in (1). Measuring mobility in terms of changes in ede income rather than welfare ensures that all indices are expressed in monetary terms, thereby providing a direct indication of the underlying scale of income movements. Accordingly, the 'cost of inequality' is defined as the income per head which could be sacrificed with no loss of welfare if the remainder were to be distributed equally:

$$
A_t = \bar{y}_t - \xi_t; \quad t = I, T
$$

(5)

where $\bar{y}_t \geq A_t \geq 0$ since $\bar{y}_t \geq \xi_t \geq 0$ by construction. This is an absolute rather than a relative measure of inequality, leading to a simple additive decomposition of total mobility into growth and redistributive elements that is independent of the order of evaluation. For the limiting case of $\varepsilon=0$, $\xi_t = W_t$ and $A_t = \int_0^1 Y_t(r)\left[1-w(r, \nu)\right] dr$ is identified as the absolute S-Gini coefficient.

In keeping with common usage in the sociological literature, structural mobility is associated with changes in the set of available income opportunities over time. We measure structural mobility as the observed change in ede incomes between the two periods:
\[ M_s = (\xi_T - \xi_1) = (\bar{Y}_T - \bar{Y}_1) + (A_1 - A_T) = M_G + M_R, \]  

which is equal, if \( v=2 \) and \( \varepsilon=0 \), to the first total mobility index proposed by Silber and Weber (2005). More generally, \( M_s \) may be seen to provide a ‘snapshot’ measure of total mobility that fully captures the welfare effects of income growth if movements of, but not within, the income distribution are of social concern. \( M_s \) may be decomposed, in the manner of van Kerm (2004) or Silber and Weber (2005), to provide a ‘growth’ mobility index, \( M_G \), due to the change in mean income over the period, and a ‘redistributive’ mobility index\(^1\), \( M_R \), due to the change in the per capita cost of inequality. \( M_R \) is invariant to equal absolute (not proportionate) growth in all incomes whereas \( M_G \) is invariant to any redistribution of a given total income among the population. \( M_s \) is a ‘directional’ measure with growth in mean incomes and reductions in cross-sectional or snapshot inequality contributing positively to the value of the index.

Corresponding to our definition of structural or snapshot mobility, we identify exchange mobility with the permutation of a fixed set of income opportunities among individuals. Thus exchange mobility is independent of structural mobility as it does not affect the level of welfare evaluated on a period-by-period basis. Nevertheless, exchange mobility is seen to be socially desirable inasmuch as it serves to attenuate initial disparities in incomes over time. The ex-ante evaluation of the incomes that individuals receive in period \( T \) based on their rank positions in period 1 yields:

\[ W_X = \int_0^1 \int_0^1 U (Y_T(q), \varepsilon) w(p, \nu) dq dp \]  

where \( W_X \) will generally exceed \( W_T \) as a result of the reranking of individuals within the income distribution. Accordingly, we measure exchange mobility as:

\[ M_X = (\xi_X - \xi_T) = (A_T - A_X), \]  

where \( \xi_X = U^{-1}(W_X) \), \( A_X = \bar{Y}_X - \xi_X \) and the second equality holds because \( \bar{Y}_X = \bar{Y}_T \). We note that if \( v=2 \) and \( \varepsilon=0 \) then \( M_X = \bar{Y} H_{AP} \), where \( H_{AP} \) is the well-known Atkinson-Plotnick reranking index which is considered by Yitzhaki and Wodon (2004) as a measure of mobility in its own right. \( M_X \) will in general be non-negative as the

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\(^1\) This term is identified as ‘structural’ mobility in Silber and Weber (2005)
concentration curve for period $T$ income (with individuals ranked on the basis of period 1 incomes) will lie on or above the Lorenz curve for period $T$ income, because for any proportion of the population $q$, the Lorenz curve records the income share of those individuals with the lowest $100q$ percent of period $T$, and not of some other group whose period 1 incomes happen to be the lowest. The index may be interpreted as a measure of the extent to which structural mobility overstates the dis-equalising effects of income growth from an ex-ante perspective due to the reshuffling of individuals within the distribution. For any given change in cross-sectional inequality, social value will be maximised ex-ante by the complete inversion of the rank ordering of income between the two periods.

The sum of structural and exchange mobility indices yields:

$$M_N = (\xi_x - \xi) = M_S + M_X = (M_G + M_R) + M_X$$

which is equal, if $v=2$ and $\varepsilon=0$, to the second total mobility index proposed by Silber and Weber (2005). More generally, $M_N$ fully captures the effects of income growth from an ex-ante perspective if individuals are risk neutral. It is an ‘ex-ante’ measure in that the evaluation of welfare in both the initial and final periods is based on the social weights associated with individuals’ ranks in the initial income distribution. This asymmetric treatment of the utility changes associated with individual income movements may be justified by a concern for the initially poor (Dardanoni, 1993), where the ‘distributional judgement’ parameter $v$ allows one to calibrate the poverty focus of the evaluation (see Essama-Nssah, 2005; Van Kerm, 2006). It is also possible in principle to employ individuals’ final period weights to evaluate mobility (see, for example, the third total mobility index proposed by Silber and Weber, 2005) but the forward-looking perspective is the more natural one when assessing the impact of mobility over time.

The risk-neutral mobility index $M_N$ may also be decomposed into vertical and (classical) horizontal mobility measures if mobility is considered as form of redistribution. Vertical mobility relates to how well individuals with different levels of initial income fare on average over time, whereas horizontal mobility has to do with the divergence in the fortunes of those with initially identical levels of income. The key to the alternative
decomposition thus rests on the identification of expected period $T$ incomes conditional upon initial income (rank) $Y_T(p) = \int_0^1 Y_t(q \mid p) dq$. The ex-ante evaluation of this conditional expected income distribution yields:

$$W_E = \int_0^1 U \left( \overline{Y}_T(p), \varepsilon \right) w(p,v) dp$$

and the corresponding ede income $\xi_E = U^{-1}(W_E)$. Hence:

$$M_N = (\xi_E - \xi) = (\overline{Y}_E - \overline{Y}_I) + (A_i - A_E) = M_v + M_H$$

where $M_v$ and $M_H$ are identified respectively as vertical and horizontal mobility indices. The former is determined by conditional expected income changes whereas the latter is determined by the conditional dispersion of income changes. Hence $M_v$ and $M_H$ are independent terms in that a change in vertical mobility may occur without affecting the level of horizontal mobility and vice versa.

The vertical index $M_v$ may be further decomposed to show how vertical mobility is determined by both the scale and pattern of income growth across the income range:

$$M_v = (\xi_E - \xi) = (\overline{Y}_E - \overline{Y}_I) + (A_i - A_E) = M_G + M_E.$$  

where $\overline{Y}_E = \overline{Y}_T$ by construction and $A_E = \overline{Y}_E - \xi_E$. Equation (12) provides a counterpart to the decomposition of $M_S$ in (5), with the common element $M_G$ providing a measure of mean income growth over the whole population as before. The vertical equity index $M_E$ provides a measure of the vertical stance of the mobility process and may be interpreted as an index of equalising opportunity in the sense of Benabou and Ok (2001). For the limiting case, $\varepsilon = 0$:

$$M_E = A_i - A_E = -\int_0^1 \left[ \overline{Y}_T(p) - Y_t(p) \right] \left[ 1 - w(p,v) \right] dp$$

which is the negative of the absolute S-concentration coefficient of conditional expected income growth calculated using initial year rankings. This in turn equals the product of the negative of the corresponding relative S-concentration coefficient, which provides an absolute counterpart of the extended Kakwani-type (1977) progressivity index of Jenkins and van Kerm (2006), and mean income growth. Thus $M_E$ will be positive if income growth is progressive (regressive) in absolute terms such that expected income gains are
a decreasing (increasing) function of initial income, and will equal zero if the growth schedule is uniform. More generally, $M_E$ will depend on both the distribution and scale of expected income growth.

The horizontal index $M_H$ will be non-positive reflecting the ex-ante welfare loss due to the classical horizontal inequity associated with the condition dispersion of future incomes. To see this point note that (7) may be re-written as:

$$W_X = \int_0^1 \int U(Y_T(q | p), \varepsilon) dq \ w(p, \nu) \ dp = \int_0^1 \bar{U}_T(p, \varepsilon) w(p, \nu) \ dp;$$

where $\bar{U}_T(p, \varepsilon)$ is expected utility in period $T$ conditional upon rank in period 1. $\xi E$ will exceed $\xi X$ if the utility function is concave and $M_H$ will therefore be non-positive. Thus the index may be interpreted as a measure of the extent to which vertical mobility overstates the positive effects of income growth because it fails to allow for the inequity manifest in the divergent fortunes of those with initially identical incomes. $M_H$ will capture both the inefficiency and the unfairness manifest in such inequity given the specification of the welfare function (cf. Broome, 1989).

Hence, if individuals are risk neutral, total mobility may be seen from an ex-ante perspective as the sum either of structural and exchange mobility or of vertical and horizontal mobility. To see the link between these alternative decompositions it is useful to characterise mobility as a stochastic process in utilities. By definition, the relationship between final and initial period utilities can be written as:

$$U_T(p, \varepsilon) = f(U_i(p, \varepsilon)) + \omega_p = \bar{U}_T(p, \varepsilon) + \omega_p; \ \forall p$$

where the growth function $f(U_i(p, \varepsilon))$ is identified as the expected value of final period utility conditional upon initial utility (rank); and $\omega_p$ is a ‘disturbance’ term having zero mean at each rank in the initial distribution. If, as is commonly assumed (see e.g. Dardanoni, 1993; Benabou and Ok, 2001), the mobility process is monotonic (implying
that $f(U_I(p, \varepsilon))$ is increasing in initial utilities) then $M_N = M_S = M_V$ if $\omega_p = 0 \forall p$,\(^2\) which provides a useful counter-factual benchmark against which to assess the consequences of a stochastic component in the mobility process. Specifically, we note that $M_N$ is invariant to noise in the utility growth process, since $W_X$ is shown in (14) to depend on conditional mean not actual utilities in period $T$, but that this is not true of $M_S$ unless $\nu = 1$ nor of $M_V$ unless $\varepsilon = 0$. In general, higher conditional dispersion in utility growth rates will be associated both with an increase in cross-sectional inequality in the final period offset by a rise in exchange mobility due to greater reranking and with more progressive income growth offset by an increase in (classical) horizontal mobility due to the lower predictability of future incomes. Exchange and horizontal mobility may thus be seen as two sides of the same coin. But, whereas the persistence of inequality over time may be seen to be more tolerable if accompanied by higher levels of exchange mobility, the pursuit of a pro-poor growth strategy will be seen to be less advantageous if accompanied by higher levels of horizontal mobility.

In contrast, Jenkins and van Kerm (2006) consider reranking to be a negative phenomenon that exacerbates cross-sectional inequality compared to what it would be if the growth process did not induce any reshuffling of individuals in the income distribution. This difference may appear purely presentational in that we could achieve a similar reversal within our framework by writing $M_S = M_N - M_X$ instead of $M_N = M_S + M_X$, which would imply $M_R = M_E - M_X$ rather than $M_E = M_R + M_X$ for the limiting case of $\varepsilon = 0$. However, the interpretation of this alternative formulation in terms of the distinct redistributive effects of income growth progressivity and reranking is problematic in that neither $M_S$ nor $M_E$ is invariant to exchange mobility – indeed a simple permutation of

\(^2\) Reranking may also arise from “utility traps” in the growth schedule, i.e. if expected utilities in period $T$ are a decreasing function of period 1 utility over some range. If the growth function is both non-stochastic and non-monotonic then $M$ will equal $M_V$ but not $M_S$. In general, the extent of any systematic reranking may be estimated as the difference between the absolute concentration coefficients of final utility ranked by expected final utility and by initial utility. For a given change in cross-sectional inequality, the presence of systematic reranking will result in the mobility process being more progressive than it would otherwise have been.
incomes must induce offsetting changes in $M_X$ and $M_e$ since cross-sectional inequality depends only on the marginal distributional of income. We would say instead that inequality in period $T$ is higher than it otherwise would be if future incomes were uniquely determined by initial income, noting that $M_e$ is invariant to a change in the conditional dispersion of individuals’ incomes even though it is sensitive to a simple permutation of incomes. Indeed, $-M_v$ is simply the difference between the absolute Ginis of predicted and actual final period income if the stochastic process is monotonic.

We further note that $M_v$ will not fully capture the full economic consequences of the unpredictability of future utilities if individuals are risk averse. We can extend our measurement framework if $0 \leq \varepsilon \leq 1$ to take account of risk aversion by considering the certainty-equivalent values of utility in period $T$. Assuming that individuals in period 1 know both their current utility and the conditional density of utility outcomes in period $T$, we define certainty-equivalent utility as:

$$
\hat{U}_T(p, \eta, \varepsilon) = \begin{cases} 
\left( \int_0^1 [U(Y_T(q | p), \varepsilon)]^{1-\eta} dq \right)^{1-\eta}, & \text{when } \eta \neq 1 \\
\exp \left( \int_0^1 \log[U(Y_T(q | p), \varepsilon)] dq \right), & \text{when } \eta = 1
\end{cases}
$$

(16)

where $\eta$ is the well known Arrow-Pratt coefficient of relative risk aversion (CRRA), with lower values of the parameter implying lower aversion to risk and the limiting case of $\eta = 0$ indicating risk neutrality. Replacing $\bar{U}_T(p, \varepsilon)$ in (10) by $\hat{U}_T(p, \eta, \varepsilon)$ yields:

$$
W_A = \int_0^1 \hat{U}_T(p, \eta, \varepsilon) w(p, \nu) dp
$$

(17)

and the corresponding edc income $\xi_A = U^{-1}(W_A)$. $\xi_A$ will be less than $\xi_X$ implying that risk averse individuals will prefer the offer of the conditional expectation $\bar{U}_T(p, \varepsilon)$ to the prospect of facing a lottery yielding the same outcome on average. Indeed, $\xi_A$ may even be lower than $\xi_T$ if the social benefits of exchange mobility are more than offset by the perceived costs of utility uncertainty. Accordingly, the risk adjustment index:

$$
M_A = (\xi_A - \xi_X)
$$

(18)

will be non-positive, providing a measure of the extent to which $M_v$ overstates the positive effects of mobility because it fails to allow for the uncertainty manifest in the
divergent fortunes of those with initially identical incomes. And it is no longer unambiguously the case that the prospect of more exchange mobility will be preferable to less holding the level of snapshot mobility $M_s$ constant (i.e. for a given change in the marginal utility distribution).

Finally, we note that all our mobility indices are expressed in monetary terms and are therefore not invariant to the choice of currency units. However all the indices may appropriately be normalised by the initial level of ede income $\xi_1$, given that the measurement framework provides an evaluation of the mobility process from the standpoint of the initial distribution of income. Moreover $M_G$, which captures the effects of income growth, may suitably be expressed as a proportion of mean income in the initial period $\bar{y}_1$. Similarly $M_R, M_X, M_E$ and $M_A$, which capture various aspects of the redistributive characteristics of the process, can be stated as a proportion of the per capita cost of inequality in the initial period $A_1$.

Estimation of mobility indices

Estimation of the mobility indices requires (sample weighted) observations on an identical sample at two points in time. $W_l$ and $W_T$ in (1) are estimated as weighted sums of sample utilities, where the utilities are calculated from (2) and the weights from (3) using the formula given in Lerman and Yitzhaki (1989) to obtain the normalised rank $r$ of each (sample weighted) observation. $M_S, M_G$ and $M_R$ are then calculated from (6) using the definitions of $\xi_t$ and $A_t$ in (4) and (5). $W_X$ is evaluated in similar fashion, cumulating final incomes over positions in the initial rather than the final income distribution, to yield an estimate of $\xi_X$ with which to calculate non-parametric estimates of $M_X$ and $M_N$ from (8) and (9) respectively.

Estimation of $M_V$ and $M_H$ is less straightforward as it requires knowledge of the conditional expectation of final incomes, $\bar{Y}_T(p)$, with which to calculate $W_E$, and hence $\xi_E$, using (10). To proceed, we note that $\bar{Y}_T(p)$ may be estimated as a function of initial incomes, i.e. $\bar{Y}_T(p) \equiv E[Y_T | Y_i(p)] = g(Y_i(p))$, using a suitable non-parametric
technique to avoid the need to specify the precise form that the function $g(\bullet)$ might take. Assuming that the conditional distribution of period $T$ incomes is lognormal, the variable span smoother of Sasieni (1998) is used to fit a local linear regression to the (sample weighted) observations on $\ln y_T$ and $y_i$ in the neighbourhood of each data point in the sample, from which estimates of the conditional expectation of final incomes $\bar{y}_T(p) = \exp(\mu_p + 0.5\sigma_p^2)$ are generated by replacing the conditional mean and variance of period $T$ log-incomes, $\mu_p$ and $\sigma_p^2$ respectively, with their sample counterparts. However a problem arises in that the resultant estimate of $\xi^E$ will not be consistent with the non-parametric estimate of $\xi^X$. To overcome this problem, we first calculate an alternative estimate of $W_x$, and hence $\xi^X$, from (14) using the local regression estimates of $\mu_p$ and $\sigma_p^2$ to calculate conditional mean utilities from:

$$U_T(p, \varepsilon) = \begin{cases} \exp\{(1 - \varepsilon)\mu_p + 0.5(1 - \varepsilon)^2\sigma_p^2\} & \text{when } \varepsilon \neq 1 \\ \mu_p & \text{when } \varepsilon = 1 \end{cases}$$

The alternative estimate of $\xi^X$ is then used in (10) to derive initial estimates of $M_V$ and $M_H$, which are scaled to sum to the non-parametric estimate of $M_N$. $M_E$ is subsequently obtained by subtraction of $M_G$ from the scaled estimate of $M_V$.

Finally, calculation of $M_A$ using (18) requires knowledge of the conditional distribution of final utilities so as to obtain mutually consistent estimates of $W_X$ and $W_A$, and hence $\xi_X$ and $\xi_A$, from (14) and (17) respectively. For this purpose, we again use our estimates of $\mu_p$ and $\sigma_p^2$ to calculate $\bar{U}_T(p, \varepsilon)$ and $\bar{U}_T(p, \eta, \varepsilon)$, where the latter is given as:

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3 Biewen (2002) has previously noted that the distribution of net equivalised household incomes in GSOEP is approximately lognormal and the assumption is more generally consistent with the positive skewness that is characteristic of income distributions. Mobility estimates based on the alternative assumption that the conditional distributions are normal with truncation from below at zero were very similar to those reported in the paper.

4 The use of a local linear regression estimator may be expected to provide a reasonable approximation to $g(\bullet)$ so long as the curvature of the unknown function is not excessive (Hastie and Loader, 1993).

5 For example, $W_E = W_X$ if $\varepsilon = 0$ but this equality will likely not hold in the estimates.
\[
\hat{U}_{T}(p, \eta, \varepsilon) \approx \begin{cases}
(\exp\left((1-\varepsilon)(1-\eta)\mu_p + 0.5(1-\varepsilon)^2(1-\eta)^2\sigma_p^2\right)\lbrack\varepsilon\rbrack^{-\eta}) & \text{when } \varepsilon \neq 1 \text{ and } \eta \neq 1 \\
\exp((1-\varepsilon)\mu(p)) & \text{when } \varepsilon \neq 1 \text{ and } \eta = 1
\end{cases}
\]

with the approximations for \( \hat{U}_{T}(p, \eta, \varepsilon) \) when \( \varepsilon = 1 \) based on second-order Taylor-series expansions about \( \mu_p \).

3. **Empirical illustration**

We apply our measurement framework to a comparative study of income mobility in the USA and Germany. The primary purpose of this study is to illustrate the use of our measures, but the application also makes a substantive contribution to the growing body of empirical literature exploring the contrasting inequality trends in the two countries (see, for example, Burkhauser and Poupore, 1997; Gottschalk and Spolaore, 2002; Massoumi and Trede, 2001; van Kerm, 2004; Jenkins and van Kerm, 2006). As is now well-established, relative inequality rose in both countries over the 1980’s and 1990’s, but by substantially more in the USA than in Germany. Our analysis allows us to assess both the nature of the income movements underlying these trends and their normative significance.

The empirical study is based on the Cross-National Equivalent File 1980-2005 (Frick et al., 2007),\(^7\) which contains equivalently defined variables for the US Panel Study of Income Dynamics (PSID) and the German Socio-Economic Panel (GSOEP). Our data for the USA covers the period 1980-97, where 1980 is the first year of PSID data in the CNEF and 1997 is the last year of data available on a consecutive annual basis. The

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\(^6\) Allowing for higher order terms results in negligible changes in the estimates of certainty equivalent utilities.

\(^7\) Further information is available from the CNEF website: http://www.human.cornell.edu/che/PAM/Research/Centers-Programs/German-Panel/cnef.cfm
German data cover the period 1984-2005, with 1984 being the first year of the GSOEP and 2005 being the latest year of GSOEP data in the CNEF, and include only observations from the original (pre-unification) West German samples. Following Jenkins and van Kerm (2006), income mobility is analysed over successive five year time spans so the US data allow eleven decompositions referring to 1981-86, 1982-87, ..., 1991-96; and the German data sixteen decompositions for 1984-89, 1985-90, ..., 1999-2004.

Our measure of income for each individual is a three-year centred moving average of post-government annual real household equivalent income calculated using the modified OECD scale.\(^8\) Averaging has commonly been used to reduce the risks of contamination of mobility estimates due to transitory income shocks and measurement error (Solon, 2002). Incomes are deflated by the consumer price index so as to reflect real purchasing power. Observations with zero or negative incomes were dropped from all samples. Sample-specific outliers were also excluded in each decomposition using the procedure outlined in Jenkins and van Kerm (2006). Sample weights and other relevant aspects of survey design are taken into account in all calculations. In particular, we take into account that the income measure is common across individuals within a household in the estimation of the conditional distributions of final income and, furthermore, that it is correlated between panel interviews in the computation of bootstrap standard errors for all statistics.

Table 1 illustrates the sensitivity of the empirical results to the choice of parameter values, presenting alternative estimates of real income mobility in the USA between 1981 and 1986. Following Duclos et al. (2003), we consider values of the relative inequality aversion parameter \(\varepsilon\) between 0 and 1, and of the ‘distributional judgement’ parameter \(\nu\) between 1 and 4. The main body of results reveals four main points of interest. First, in the absence of both rank dependence \((\nu = 1)\) and inequality aversion \((\varepsilon = 0)\) then the risk-

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\(^8\) Using annual incomes rather than three-year moving averages yields broadly similar results, but with predictably higher levels of both exchange and horizontal mobility, resulting in an increase in the index of equalising opportunities given that redistributive mobility is roughly the same, and a larger risk adjustment.
neutral mobility index $M_N$ is simply equal to mean income growth $M_G$, which was $2275$ in constant 1990 prices.

Table 1. Real income mobility in the USA, 1981-1986 (Dollars at constant 1990 prices)

<table>
<thead>
<tr>
<th>Distributional judgement</th>
<th>$\varepsilon$</th>
<th>0</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>0.5</th>
<th>1</th>
<th>0.5</th>
<th>1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial:</td>
<td>$\xi_i$</td>
<td>14521</td>
<td>10464</td>
<td>7923</td>
<td>13600</td>
<td>12711</td>
<td>9970</td>
<td>7620</td>
<td>9473</td>
<td>7314</td>
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<tr>
<td>$\bar{y}_i$</td>
<td>14521</td>
<td>14521</td>
<td>14521</td>
<td>14521</td>
<td>14521</td>
<td>14521</td>
<td>14521</td>
<td>14521</td>
<td>14521</td>
<td>14521</td>
</tr>
<tr>
<td>$A_i$</td>
<td>0</td>
<td>4058</td>
<td>6598</td>
<td>921</td>
<td>1810</td>
<td>4551</td>
<td>6901</td>
<td>5049</td>
<td>7207</td>
<td></td>
</tr>
<tr>
<td>Final:</td>
<td>$\xi_f$</td>
<td>16797</td>
<td>11635</td>
<td>8475</td>
<td>15494</td>
<td>14245</td>
<td>10949</td>
<td>8059</td>
<td>10257</td>
<td>7645</td>
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<tr>
<td>$\bar{y}_f$</td>
<td>16797</td>
<td>16797</td>
<td>16797</td>
<td>16797</td>
<td>16797</td>
<td>16797</td>
<td>16797</td>
<td>16797</td>
<td>16797</td>
<td>16797</td>
</tr>
<tr>
<td>$A_f$</td>
<td>0</td>
<td>5162</td>
<td>8322</td>
<td>1302</td>
<td>2552</td>
<td>5848</td>
<td>8738</td>
<td>6540</td>
<td>9151</td>
<td></td>
</tr>
</tbody>
</table>

| Structural mobility | $M_s$ | 2275 | 1172 | 552 | 1894 | 1534 | 979 | 439 | 785 | 331 |
| Growth | $M_G$ | 2275 | 2275 | 2275 | 2275 | 2275 | 2275 | 2275 | 2275 | 2275 |
| Redistribution | $M_E$ | 0 | -1104 | -1724 | -381 | -741 | -1297 | -1837 | -1491 | -1944 |
| Exchange mobility | $M_x$ | 0 | 1047 | 1528 | 0 | 0 | 885 | 1320 | 742 | 1125 |

| Risk neutral mobility | $M_N$ | 2275 | 2218 | 2080 | 1894 | 1534 | 1864 | 1579 | 1456 | 1456 |
| Vertical mobility | $M_V$ | 2275 | 2218 | 2080 | 2484 | 2676 | 2309 | 2139 | 2401 | 2199 |
| Growth | $M_G$ | 2275 | 2275 | 2275 | 2275 | 2275 | 2275 | 2275 | 2275 | 2275 |
| Equalizing opp. | $M_E$ | 0 | -57 | -196 | 209 | 400 | 34 | -136 | 125 | -77 |
| Horizontal mobility | $M_H$ | 0 | 0 | 0 | -590 | -1142 | -445 | -380 | -747 | -473 |

| Risk adjustment | $M_a$ | $\eta = 0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\eta = 0.5$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\eta = 1$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\eta = 2$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\eta = 4$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\eta = 8$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

15
Second, rank dependence alone gives rise to redistributive mobility $M_R$, due to the change in cross-sectional inequality as measured by the S-Gini index, and exchange mobility $M_X$, due to the reshuffling of individuals within the income distribution, but not to classical horizontal inequities so $M_H$ equals zero. The size of both $M_R$ and $M_X$ are increasing in the social weight attached to the fortunes of the initially poor. Holding the poverty focus constant and increasing the level of inequality aversion reduces the size of $M_X$ due to the concavity of the utility function.

Third, inequality aversion alone gives rise to redistributive mobility $M_R$, due to the change in cross-sectional inequality as measured by the Atkinson index, and horizontal inequity mobility $M_H$, due to the divergent fortunes of initially equal individuals, but not to reranking effects so $M_H$ equals zero. Both $M_R$ and $M_H$ are increasing in the degree of inequality aversion as the inefficiency associated with both the unconditional and conditional dispersion of incomes rises. Holding the level of inequality aversion fixed and increasing the poverty focus of the evaluation reduces the size of $M_H$ given that the conditional dispersion of incomes is greater among the rich than the poor.

Finally, the index of equalising opportunity $M_E$ is equal to the difference between the cost of inequality associated with the distribution of actual incomes in the initial period and the distribution of expected incomes in the final period. Accordingly, the index may be interpreted as a residual term that reflects the balance between $-M_R$ on the one hand and $M_X$ and $-M_H$ on the other. A sufficient condition for mobility to be deemed “progressive” in absolute terms is that the positive ex-ante value of exchange mobility exceed any increase in cross-sectional inequality. $M_E$ is more positive, or less negative, the higher the degree of inequality aversion and, hence, the more rapidly the marginal utility of income diminishes with income.

Table 1 also reports the risk adjustment index $M_A$ for values of the relative risk aversion parameter $\eta$ between 0 and 8 to cover the very wide range of CRRA values reported in the literature (see e.g. Kaplow, 2005; Meyer and Meyer, 2006). $M_A$ increases with the degree of risk aversion, exceeding both $M_X$ and $M_E$ for sufficiently high values of $\eta$. By
implication, whether or not exchange and overall mobility are perceived to be desirable
social phenomena from an ex-ante perspective may well depend on public attitudes to
income uncertainty.

Table 2 presents the main set of results for the USA, based on intermediate values of
$\varepsilon=0.5$ and $v=2$, implying moderate levels of inequality aversion and concern for the poor,
and the full range of CRRA values. The cross-sectional statistics indicate that both mean
income and absolute inequality rose in real terms in each 5 year sub-period between 1981
and 1996, with the rise in the former exceeding the latter in all sub-periods except for two
that span the recession of the early 1990's. Thus the analysis of structural mobility shows
that the growth index $M_G$ is consistently positive and the redistribution index $M_R$
negative, with the overall index of snapshot mobility $M_S$ showing that welfare evaluated
on a period-by-period basis rose in all but two of the five year spans. The positive
association between the magnitudes of $M_G$ and $M_R$ suggests that the nature of economic
growth in the USA was inherently disequalising, with both mean incomes and absolute
inequality rising more rapidly during economic upswings.

However, the structural analysis of mobility does not take the movement of individuals
within the income distribution into account and may therefore overstate social
perceptions of the disequalising effects of income growth over the period. From an ex-
ante risk-neutral perspective, welfare rose in each sub-period as indicated by the
consistently positive values of the mobility index $M_N$. By implication, the prospect of the
income distribution five years hence was consistently preferable to the current
distribution, evaluated on the basis of individuals’ ranking in the current distribution.
The level of exchange mobility $M_X$ was roughly constant over the whole period,
suggesting that the processes leading to the reranking of individuals were largely
unaffected by the strength of the economy. Indeed, if anything, exchange mobility
appears to be a mildly counter-cyclical phenomenon. This view is reinforced by
consideration of the horizontal mobility index $M_H$ from the alternative decomposition of
$M_N$. Thus $M_H$, which is determined by the conditional dispersion of income prospects
over each five year time span, similarly lies within a relatively narrow range over the
Table 2. Real income mobility in the USA over successive 5 year spans, 1981-1996 (US Dollars at constant 1990 prices): $\varepsilon=0.5$, $\nu=2$.

<table>
<thead>
<tr>
<th>Period</th>
<th>81-86</th>
<th>82-87</th>
<th>83-88</th>
<th>84-89</th>
<th>85-86</th>
<th>86-91</th>
<th>87-92</th>
<th>88-93</th>
<th>89-94</th>
<th>90-95</th>
<th>91-96</th>
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<td>Initial:</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ede income</td>
<td>$\xi_i$</td>
<td>9966</td>
<td>9993</td>
<td>10177</td>
<td>10330</td>
<td>10565</td>
<td>10819</td>
<td>11142</td>
<td>11333</td>
<td>11507</td>
<td>11580</td>
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<tr>
<td></td>
<td>$\bar{y}_i$</td>
<td>14481</td>
<td>14709</td>
<td>15199</td>
<td>15575</td>
<td>16163</td>
<td>16599</td>
<td>17260</td>
<td>17622</td>
<td>18028</td>
<td>18270</td>
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<td>absolute inequality $A_i$</td>
<td>4515</td>
<td>4716</td>
<td>5021</td>
<td>5245</td>
<td>5598</td>
<td>5780</td>
<td>6119</td>
<td>6289</td>
<td>6521</td>
<td>6689</td>
<td>6923</td>
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<td>Final:</td>
<td></td>
<td></td>
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<td></td>
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<td>ede income</td>
<td>$\xi_f$</td>
<td>10951</td>
<td>11227</td>
<td>11457</td>
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<td>11515</td>
<td>11461</td>
<td>11632</td>
<td>11539</td>
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<td>absolute inequality $A_f$</td>
<td>5812</td>
<td>6058</td>
<td>6309</td>
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<td>7263</td>
<td>7451</td>
</tr>
<tr>
<td>Growth $M_o$</td>
<td>2281</td>
<td>2575</td>
<td>2568</td>
<td>2506</td>
<td>2068</td>
<td>1671</td>
<td>1203</td>
<td>847</td>
<td>346</td>
<td>146</td>
<td>801</td>
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<td>Redistribution $M_s$</td>
<td>-1297</td>
<td>-1342</td>
<td>-1288</td>
<td>-1119</td>
<td>-1029</td>
<td>-642</td>
<td>-346</td>
<td>-567</td>
<td>-273</td>
<td>-104</td>
<td>-81</td>
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<td>Structural mobility $M_{s_s}$</td>
<td>984</td>
<td>1234</td>
<td>1280</td>
<td>1158</td>
<td>949</td>
<td>642</td>
<td>490</td>
<td>205</td>
<td>-180</td>
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<td>273</td>
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<td>Exchange mobility $M_x$</td>
<td>875</td>
<td>909</td>
<td>915</td>
<td>918</td>
<td>901</td>
<td>861</td>
<td>872</td>
<td>901</td>
<td>939</td>
<td>990</td>
<td>947</td>
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<td>Risk neutral mobility $M_n$</td>
<td>1859</td>
<td>2143</td>
<td>2195</td>
<td>2076</td>
<td>1850</td>
<td>1503</td>
<td>1362</td>
<td>1106</td>
<td>759</td>
<td>562</td>
<td>1220</td>
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<td>Vertical mobility $M_v$</td>
<td>2294</td>
<td>2592</td>
<td>2664</td>
<td>2556</td>
<td>2318</td>
<td>1987</td>
<td>1882</td>
<td>1664</td>
<td>1420</td>
<td>1230</td>
<td>1575</td>
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<tr>
<td>Growth $M_o$</td>
<td>2281</td>
<td>2575</td>
<td>2568</td>
<td>2506</td>
<td>2068</td>
<td>1671</td>
<td>1203</td>
<td>847</td>
<td>346</td>
<td>146</td>
<td>801</td>
</tr>
<tr>
<td>Equalizing opp. $M_e$</td>
<td>12</td>
<td>17</td>
<td>95</td>
<td>50</td>
<td>250</td>
<td>316</td>
<td>679</td>
<td>816</td>
<td>1074</td>
<td>1084</td>
<td>954</td>
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<tr>
<td>Horizontal mobility $M_{H_i}$</td>
<td>-434</td>
<td>-450</td>
<td>-468</td>
<td>-480</td>
<td>-485</td>
<td>-520</td>
<td>-557</td>
<td>-661</td>
<td>-669</td>
<td>-535</td>
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<td>Risk adjustment: $M_a$</td>
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<tr>
<td>$\eta = 0$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<td>$\eta = 0.5$</td>
<td>-198</td>
<td>-207</td>
<td>-211</td>
<td>-213</td>
<td>-206</td>
<td>-205</td>
<td>-216</td>
<td>-229</td>
<td>-237</td>
<td>-245</td>
<td>-225</td>
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<tr>
<td>$\eta = 2$</td>
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<td>-805</td>
<td>-821</td>
<td>-831</td>
<td>-803</td>
<td>-800</td>
<td>-840</td>
<td>-889</td>
<td>-919</td>
<td>-948</td>
<td>-875</td>
</tr>
<tr>
<td>$\eta = 4$</td>
<td>-1489</td>
<td>-1555</td>
<td>-1585</td>
<td>-1605</td>
<td>-1552</td>
<td>-1546</td>
<td>-1621</td>
<td>-1712</td>
<td>-1767</td>
<td>-1821</td>
<td>-1689</td>
</tr>
<tr>
<td>$\eta = 8$</td>
<td>-2786</td>
<td>-2907</td>
<td>-2962</td>
<td>-2997</td>
<td>-2905</td>
<td>-2892</td>
<td>-3023</td>
<td>-3178</td>
<td>-3270</td>
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<td>-3155</td>
</tr>
<tr>
<td>Note: Bootstrap standard errors in parentheses based on 200 replications.</td>
<td></td>
<td></td>
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<td></td>
<td></td>
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</tr>
</tbody>
</table>
whole study period but is significantly higher in those sub-periods spanning the slowdown of the early 1990’s. We conclude that future incomes were less predictable in the USA during slowdowns given that both exchange and horizontal mobility may be seen to arise from the stochastic nature of the mobility process.

The results of the alternative decomposition also show that distribution of expected income opportunities in the final year of each sub-period was consistently equalising from an ex-ante welfare perspective, as indicated by the positive values of $M_E$, with income mobility more “progressive” in absolute terms during economic slowdowns. These findings appear to conflict with the evidence that cross-sectional inequality was increasing throughout the whole period, but may be reconciled once exchange and horizontal mobility are taken into account. Jenkins and van Kerm (2006) have explained in detail how pro-poor income growth can be accompanied by rising inequality due to the reshuffling of individuals within the income distribution over time. Measuring inequality in relative terms, they report that exchange mobility exceeded the rise in inequality in the USA throughout the 1980’s. In contrast, measuring inequality in absolute terms, we find exchange mobility $M_X$ to have only exceeded the disequalising effects of redistributive mobility $M_R$ in the latter sub-periods spanning the slowdown of the early 1990’s. However, within our framework, costs of inequality may also arise from the classical horizontal inequities associated with the divergence in individual income changes about conditional mean growth rates. Thus income mobility would be $M_Y$ rather than $M_N$, evaluated ex-ante on a risk-neutral basis, if all individuals received conditional mean incomes in the final year of each sub-period, where the former (but not the latter) exceeds $M_G$ in all sub-periods. Figure 1 displays the concentration curves of the utility changes associated with conditional mean income growth, with individuals ranked in ascending

---

9 If $\varepsilon$ is set equal to zero then mobility is deemed to be progressive (i.e. $M_X + M_R > 0$ since $M_H = 0$ in this case) in all sub-periods from 1985-90 onwards, providing a direct counterpart of the Jenkins and van Kerm (2006) results but with inequality measured in absolute rather than relative terms.
order of utility in the initial year, for 1981-86, 1986-91 and 1991-96. All three curves lie above the line of perfect equality indicating that opportunities for utility growth were consistently progressive in absolute terms from an ex-ante perspective. Moreover, the three curves lie ever further above the line of perfect equality indicating that the absolute progressivity of the mobility process was negatively associated with income growth which was weaker in each successive sub-period. Indeed, it is evident from the concentration curve for 1991-96 that the conditional mean income growth of many of those in the top income quintile in 1991 was negative over this five year time span.

Figure 1. Concentration curves of utility changes associated with conditional mean income growth ordered by utility in the initial year: USA 1981-86, 1986-91 and 1991-96.

Note that $M_E$ can not in general be written as a weighted sum of the utility changes associated with individual income movements. Nevertheless, the Figure does serve to corroborate the positive values obtained for $M_E$ by illustrating the progressivity of individual utility changes in absolute terms.
Finally, the estimates of the risk adjustment index $M_\alpha$ in Table 2 serve as a reminder that social perceptions of income mobility are sensitive to attitudes to risk. In particular, the ex-ante welfare loss due to risk aversion would have exceeded the perceived benefits of exchange mobility, holding the level of snapshot mobility $M_S$ constant, if the CRRA had been greater than about two. Overall, mobility would not have been perceived ex-ante as resulting in a loss of welfare unless the CRRA had been appreciably greater than four during the economic recovery of the early 1980’s, but this value drops to only about one in the slowdown of the early 1990’s.

Table 3 presents comparable results for Germany for the period 1985-2004, from which it emerges that the nature of the mobility process was broadly similar to that in the USA. First, income growth was disequalising in absolute terms, with cross-sectional income inequality actually falling during the post-unification recession of the early 1990’s. However, Figure 2 shows that the trade-off between income growth and inequality was more favourable in Germany than the USA, with the lower trend line for Germany indicating that a given proportionate change in mean income was associated with a smaller proportionate change in absolute inequality in Germany. That most points lie above the 45° line implies that relative inequality rose in virtually all sub-periods in both countries.

Second, the degree of unpredictability of future incomes appears largely insensitive to the state of the economy as indicated by the relatively stable values of $M_X$ and $M_H$. However, there is some evidence of pro-cyclicality, unlike the USA, perhaps in part as a result of stronger social insurance mechanisms to protect incomes during recessions. Figure 3 shows that the trade-off between vertical and horizontal mobility was more favourable in Germany than the USA, with the lower trend line for Germany indicating that a given proportionate change in ede income due to the conditional mean growth of incomes was associated with a smaller proportionate loss due to the horizontal inefficiencies associated with the conditional dispersion of final year incomes. In the absence of horizontal mobility, ede income in the final year of each sub-period would have been between 2.1% and 3.1% higher in Germany and between 3.8% and 5.7% higher in the USA. However,
<table>
<thead>
<tr>
<th>Period</th>
<th>85-90</th>
<th>86-91</th>
<th>87-92</th>
<th>88-93</th>
<th>89-94</th>
<th>90-95</th>
<th>91-96</th>
<th>92-97</th>
<th>93-98</th>
<th>94-99</th>
<th>95-00</th>
<th>96-01</th>
<th>97-02</th>
<th>98-03</th>
<th>99-04</th>
</tr>
</thead>
</table>
| **Initial:**
| \( \xi_i \) | (102) | (111) | (120) | (119) | (137) | (142) | (133) | (177) | (159) | (234) | (184) | (185) | (232) | (206) | (213) |
| \( \bar{y}_i \) | 14542 | 14786 | 15172 | 15710 | 16130 | 16630 | 17112 | 17586 | 17809 | 17469 | 17738 | 17763 | 17300 | 17312 | 17334 |
| \( A_i \) | 3542 | 3660 | 3742 | 3846 | 3998 | 4129 | 4307 | 4622 | 4735 | 4730 | 4793 | 4881 | 4749 | 4661 | 4658 |
| Final:
| \( \xi_f \) | 12669 | 12816 | 12982 | 12954 | 12756 | 12733 | 12734 | 12745 | 12884 | 12996 | 13455 | 13330 | 13367 | 13359 | 13112 |
| \( \bar{y}_f \) | 16851 | 17194 | 17544 | 17436 | 17385 | 17280 | 17125 | 17290 | 17568 | 17778 | 18451 | 18560 | 18949 | 18482 | 18272 |
| \( A_f \) | 4182 | 4379 | 4562 | 4482 | 4629 | 4547 | 4390 | 4544 | 4684 | 4782 | 4995 | 5230 | 5127 | 5124 | 5160 |
| \( M_s \) | 1669 | 1690 | 1552 | 1091 | 624 | 232 | -71 | -219 | -190 | 257 | 511 | 448 | 816 | 708 | 436 |
| \( M_o \) | (88) | (94) | (118) | (103) | (142) | (125) | (114) | (112) | (120) | (136) | (131) | (150) | (95) | (105) | (158) |
| \( M_r \) | (119) | (122) | (124) | (123) | (133) | (127) | (143) | (148) | (133) | (157) | (169) | (216) | (184) | (227) | (229) |
| \( M_e \) | 963 | 1034 | 996 | 1003 | 999 | 990 | 915 | 891 | 835 | 873 | 891 | 1020 | 994 | 1086 | 1011 |
| \( M_n \) | (45) | (48) | (52) | (56) | (51) | (50) | (53) | (60) | (54) | (49) | (51) | (75) | (70) | (100) | (122) |
| **Risk neutral mobility**
| \( M_s \) | 2632 | 2723 | 2548 | 2094 | 1624 | 1222 | 844 | 672 | 645 | 1130 | 1402 | 1467 | 1809 | 1794 | 1448 |
| \( M_o \) | (94) | (101) | (128) | (112) | (136) | (133) | (126) | (140) | (131) | (142) | (126) | (159) | (169) | (260) | (213) |
| \( M_r \) | 2936 | 3047 | 2882 | 2425 | 1976 | 1564 | 1177 | 992 | 964 | 1439 | 1700 | 1835 | 2168 | 2222 | 1817 |
| \( M_e \) | 627 | 638 | 510 | 699 | 721 | 914 | 1164 | 1289 | 1205 | 1130 | 987 | 1039 | 975 | 1051 | 879 |
| \( M_n \) | (96) | (89) | (90) | (95) | (98) | (100) | (106) | (118) | (103) | (121) | (122) | (149) | (147) | (173) | (146) |
| **Horizontal mobility**
| \( M_s \) | -304 | -323 | -334 | -331 | -353 | -324 | -333 | -320 | -319 | -308 | -298 | -368 | -359 | -428 | -369 |

| Risk adjustment: \( \eta \) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| \( \eta = 0 \) | -146 | -158 | -161 | -157 | -163 | -157 | -157 | -153 | -142 | -142 | -141 | -173 | -164 | -190 | -173 |
| \( \eta = 0.5 \) | (7) | (8) | (7) | (8) | (9) | (10) | (9) | (11) | (9) | (10) | (8) | (12) | (13) | (17) | (18) |
| \( \eta = 1 \) | -291 | -314 | -319 | -313 | -324 | -312 | -313 | -304 | -282 | -283 | -281 | -343 | -327 | -377 | -343 |
| \( \eta = 8 \) | (27) | (31) | (28) | (33) | (35) | (38) | (34) | (42) | (33) | (40) | (30) | (46) | (48) | (64) | (71) |

Note: Bootstrap standard errors in parentheses based on 200 replications.
Figure 2. Growth of mean incomes and cross-sectional inequality.

Figure 3. Changes in welfare due to vertical and horizontal mobility.

Figure 4. Changes in welfare due to equalising opportunities and mean income growth.
lower levels of horizontal mobility in Germany did not translate into comparably lower levels of exchange mobility, which averaged about 8% of initial year ede income in both countries, because of lower levels of absolute inequality and more pro-poor income growth (see below).

Third, the index of equalising opportunity $M_e$ is consistently positive, with mobility more “progressive” in economic downturns. Figure 4 shows that the trade-off between vertical equity and mean income growth was more favourable in Germany than the USA, with the higher trend line for Germany at high levels of mean income growth indicating that a given proportionate change in ede income due to mean income growth was associated with a larger proportionate rise due to vertical redistribution during economic upswings. We conclude that the income growth process in Germany was generally more pro-poor than in the USA.

Finally we note that the threshold CRRA values at which the risk adjustment index outweigh the levels of exchange and overall mobility are higher in Germany than the USA, reflecting the lower levels of income uncertainty.

4. Conclusion
The paper proposes a framework that can be used to both characterise and quantify the welfare effects of income mobility from an ex-ante, risk-neutral perspective. The resultant class of measures can be decomposed not only in terms of structural and exchange mobility but also in terms of vertical and horizontal mobility, thereby encompassing two of the main approaches to the economic evaluation of mobility currently found in the literature. We further show how the framework can be extended to take account of risk aversion by assuming that individuals consider some certainty equivalent when evaluating the set of utility opportunities that they face. All measures are expressed in monetary terms and therefore provide a direct indication of the underlying scale of income movements.

Our analysis reveals a tension between the perceived value of exchange mobility on the one hand and the horizontal inequities associated with the conditional dispersion of future
incomes on the other. Given this tension, whether the stochasticity of the mobility process is seen as being desirable or not is likely to depend on the political culture of a society which in turn will shape the choice of policies that determine the future evolution of the income distribution. Alesina et al. (2004) argue that the greater tolerance of inequality that is evident amongst the poor in America compared to Europe can be explained by their perception that they are living in a society with higher opportunities for (upward) mobility. Conversely, more equal societies may be expected to put more weight on the predictability of incomes given that income differences between social classes are less pronounced. Adsera and Boix (2000) characterise continental European models of social democracy in terms of the compression of wage differentials in conjunction with high levels of social insurance compared to the USA, UK and Japan.

Our empirical results show that levels of both structural and vertical mobility were directly linked to the strength of the economy in both the USA and Germany, but that levels of exchange and horizontal mobility, which are primarily determined by the predictability of future incomes, were much less sensitive to the economic cycle. The pattern of income mobility in the USA has been both less pro-poor than in Germany and more horizontally inequitable, but the latter did not translate into higher levels of exchange mobility given the vertical stance of the growth process and higher levels of absolute inequality. Thus, like the majority of studies on the issue (see, for example, Jenkins and van Kerm, 2006), we find there to have been less exchange mobility in the USA than Germany, but our measurement framework also provides fresh insight into why, contrary to popular belief, this may have been the case.

However it would be unwise to try to read too much into the empirical results. For a start, the analysis in each sub-period is based on a balanced panel and income differentials may be expected to have changed over the five year time span simply due to the ageing of the sample (Ayala and Sastre, 2002). Moreover, the estimates of the mobility indices may be biased due to non-random sample attrition over each sub-period (Frick et al., 2007). More generally, there is a need for more research into the determinants of individual income dynamics in order to better explain the sources of observed changes in cross-section or snapshot inequality over time.
References


