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Abstract

We study the asymmetric and dynamic dependence between financial assets and demonstrate, from the perspective of risk management, the economic significance of dynamic copula models. First, we construct stock and currency portfolios sorted on different characteristics (ex ante beta, coskewness, cokurtosis and order flows), and find substantial evidence of dynamic evolution between the high beta (respectively, coskewness, cokurtosis and order flow) portfolios and the low beta (coskewness, cokurtosis and order flow) portfolios. Second, using three different dependence measures, we show the presence of asymmetric dependence between these characteristic-sorted portfolios. Third, we use a dynamic copula framework based on Creal et al. (2013) and Patton (2012) to forecast the portfolio Value-at-Risk of long-short (high minus low) equity and FX portfolios. We use several widely used univariate and multivariate VaR models for the purpose of comparison. Backtesting our methodology, we find that the asymmetric dynamic copula models provide more accurate forecasts, in general, and, in particular, perform much better during the recent financial crises, indicating the economic significance of incorporating dynamic and asymmetric dependence in risk management.

Key words: asymmetric dependence, dynamic copulas, tail risk, Value-at-Risk forecasting.

JEL codes: C32, C53, G17, G32

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1 Introduction

Recent financial crises have highlighted the need for a deeper understanding of the dynamics and asymmetry of the dependence structure between financial assets and more reliable quantitative measures to forecast risk.

We bring together two strands of literature. The first strand is related to the phenomenon, termed asymmetric dependence, whereby the returns on two assets exhibit greater correlation, or more generally, greater dependence during market downturns than market upturns. The second strand constructs characteristic-sorted portfolios. We sort US and UK stocks into portfolios using the ex ante beta of individual stocks and their coskewness and cokurtosis with the market, reflecting their well-known importance in pricing securities (see Harvey and Siddique, 2000; Bakshi et al., 2003; Conrad et al., 2013). Moreover, it is well known from the literature that order flow can explain the contemporaneous returns of financial assets (see Evans and Lyons, 2002; Brandt and Kavajecz, 2004), and predict future exchange returns (see Menkhoff et al., 2013). In our study, we sort currencies into portfolios based on the signs and magnitude of aggregated order flow (as a proxy for market pressure).

Bringing together the two strands, we examine the dependence structure between the high portfolios (i.e. portfolios with the highest beta, coskewness, cokurtosis and total order flow) and the low portfolios (i.e. portfolios with the lowest ex ante beta, coskewness, cokurtosis and total order flow).

Our paper makes four main contributions. First, we provide a comprehensive study of the dynamic evolution of dependence in both equity markets and foreign exchange (FX) markets. We find evidence that the dependence structures between characteristic-sorted portfolios, such as the high beta portfolio and the low beta portfolio, significantly changed after the start of the global financial crisis of 2007-2009. Second, we provide new empirical evidence of asymmetric dependence in the US and UK equity markets. In general, we show that the coefficients of lower tail dependence (LTD) are greater than the coefficients of upper tail dependence (UTD) and that this asymmetry is statistically significant. This finding is obviously important for the hedging of risk and for portfolio management. We also show that compared with the UK portfolios, the US portfolios are not only more crash sensitive during market downturns, but also more boom sensitive during market upturns. Third, while estimation of portfolio VaR has been widely studied in the literature, there have been relatively few studies examining portfolio VaR forecasting, especially forecasting through dynamic copulas. Using the characteristic-sorted portfolios, we evaluate the economic
significance of incorporating asymmetric and dynamic dependence into VaR forecasts, and we empirically show that dynamic copula models can actually improve the portfolio Value-at-Risk (VaR) predictions. Our backtesting results provide solid evidence that dynamic copula models (based on the Generalized Autoregressive Score (GAS) models of Creal et al., 2013) can consistently provide better VaR forecasts than alternative benchmark models, especially at the 99% level. And we also find that semiparametric dynamic copula models perform relatively better than full parametric dynamic copula models. Fourth, to the best of our knowledge, this paper is the first study using long-short (high minus low) portfolio returns to forecast VaR. This is important for the following reasons: First, long-short portfolio returns have different properties compared to simple long or short return series; Second, long-short portfolios have recently become increasingly popular in studies of asset pricing; Third, modeling the VaR of long-short portfolios is of interest to practitioners as long-short strategies are widely used in the financial industry.

One feature of recent financial crises is the extent to which financial assets that had previously behaved mostly independently suddenly moved together. This phenomenon is usually termed asymmetric dependence, see for instance Longin and Solnik (2001), Ang and Bekaert (2002), Ang and Chen (2002), Poon et al. (2004), Patton (2006), Okimoto (2008) and Christoffersen and Langlois (2013). The presence of asymmetric correlations is obviously important, as it can cause serious problems in hedging effectiveness and portfolio diversification (see Hong et al., 2007). In the foreign exchange (FX) markets, Patton (2006) suggests that this asymmetry is possibly caused by the asymmetric responses of central banks to exchange rate movements. In the equity markets, although there have been many studies of asymmetric dependence, there is no consensus on the underlying economic cause. One possible cause is that risk-averse investors treat downside losses and upside gains distinctively, which is consistent with “Prospect Theory” (see Kahneman and Tversky, 1979).

Clearly, the key to portfolio risk management is to recognize how quickly and dramatically the dependence structure can change. An increasingly popular method for constructing high dimensional dependence is based on copulas. Copulas are functions that connect multivariate distributions to their one-dimensional marginals (Sklar, 1959). The copula approach is particularly useful in portfolio risk measurement for the following reasons. First, copulas can describe the dependence between assets under extreme circumstances, as they use a quantile scale. Second, they utilize a flexible bottom-up approach that can combine a variety of marginal models with a variety of possible dependence specifications (McNeil et al., 2005). Ideally, an appropriate copula for financial modeling should be capable of accommodating both
positive and negative dependence, capturing both symmetric and asymmetric dependence, and allowing for the possible tail dependence. The skew $t$ copula of Demarta and McNeil (2005) can be viewed as a flexible extension that contains all these desirable properties.

The econometrics literature provides a wealth of evidence that the conditional dependence structure between assets varies through time (see Giacomini et al., 2009; Rémillard, 2010). This noteworthy phenomenon motivates the consideration of dynamic copula models which allow the correlation parameter to change dynamically. One such model is proposed by Patton (2006) who extended Sklar’s theorem for conditional distributions and proposed an observation driven conditional copula model. This model defined the time-varying dependence parameter of a copula as a parametric function of transformations of the lagged data and an autoregressive term. Another example is the dynamic conditional correlation (DCC) model proposed by Engle (2002). For instance, Christoffersen et al. (2012) and Christoffersen and Langlois (2013) developed a dynamic asymmetric copula (DAC) model based on the DCC model to capture long-run and short-run dependence, multivariate nonnormality, and dependence asymmetries.

Christoffersen and Langlois (2013) investigate dependence between portfolios sorted on the Fama-French and momentum factors. Our study extends their research. We focus on portfolios sorted on different moment characteristics of individual securities (ex ante beta, conditional skewness and conditional kurtosis), as earlier empirical studies provide evidence that a stock’s ex ante beta, co-skewness and co-kurtosis with the market portfolio are economically important in asset pricing. For instance, Harvey and Siddique (2000) show the usefulness of conditional skewness (or coskewness) in explaining the cross-sectional variation of securities returns. Their results show that stocks with negative ex ante coskewness (left-skewed) tend to command higher equilibrium risk compensations. Moreover, they point out that coskewness is able to capture the downside risk, which is the main concern in risk management. Dittmar (2002) tests whether investors’ expected returns are influenced by stock’s coskewness and cokurtosis with the market portfolio. In our study, we sort US and UK stocks into portfolios using the ex ante beta of individual stocks and their coskewness and cokurtosis with the market. Our FX portfolios are sorted on aggregated order flow, which can be used to explain contemporaneous returns and predict future returns (see Evans and Lyons, 2002; Brandt and Kavajecz, 2004; Menkhoff et al., 2013) By doing so, we can investigate the variation of dependence structure between the portfolios with different characteristics in different market conditions.

Creal et al. (2013) proposed a class of Generalized Autoregressive Score (GAS) models, which use
the scaled score of a likelihood function to update the parameters over time. The GAS model is a consistent and unified framework, which encompasses some well-known models including the GARCH, the autoregressive conditional duration, the autoregressive conditional intensity, and Poisson count models with time-varying mean. They illustrate the GAS framework by introducing a new model specification for a dynamic copula\(^1\). Based on simulation results and empirical evidence, they point out that the driving mechanism in Patton (2006) only captures some of the changes in the dependence coefficients. Specifically, it has shortcomings in tracking the upper and lower tail dependence dynamics simultaneously, since the constant mechanism applies to both types of dependence. Conversely, the GAS specification has better performance in capturing different types of dynamics. Thus, our study adopts it as the driving mechanism to update copula parameters.

VaR, which measures the losses, at a given significance level, over a specific time horizon, is a routinely used method for the valuation of market risk (see Jorion, 2007, for a comprehensive survey). Financial institutions are allowed to develop internal models to calculate their own VaR. In addition, margin requirements, counterparty exposures and collateral requirements are normally calculated by methods based on the estimation of VaR. Thus, financial institutions have a strong motivation to develop more accurate internal VaR models to predict and prevent risk. Recent financial crises have further revealed numerous weaknesses in banks’ existing VaR models and demonstrated the need for improvement in portfolio VaR methodologies. In this paper, we apply the dynamic copula framework proposed by Creal et al. (2013) and Patton (2012) to VaR prediction. Backtesting is used to evaluate the performance of our models and benchmarks.

The remainder of this paper is organized as follows. In Section 2, we detail the methods we employ for portfolio sorting, and we provide an overview of copula theory and three computation methods for tail dependence coefficients. Then, we present the dynamic copula model and its estimation methodology for risk management. The data used in the paper, summary statistics and univariate model estimations are in Section 3. In Section 4, we focus on testing whether the dependence structures between characteristic-sorted portfolios are statistically dynamic and asymmetric, especially during the global financial crisis of 2007-2009 and the Euro Sovereign Debt crisis of 2010-2011, and then discuss the possible reasons for different kinds of dependence. In Section 5, we predict ex ante portfolio Value-at-Risk using dynamic

\(^1\)Harvey (2013) proposes a similar approach for modeling time-varying parameters, which he calls a “dynamic conditional score (DCS)” model.
copulas and 10 benchmark models and report the comparison results of backtesting. Finally, conclusions are given in Section 6. All the tables and figures used in this paper are presented in the Appendix.

2 Model Specification

In this section, we detail the models and portfolio construction that we use in this paper.

2.1 Portfolio Construction

The return on an asset is defined as the first difference of the log price, \( r_t = \log P_t - \log P_{t-1} \).

We consider now the equity portfolios. We construct portfolios separately sorted on ex ante beta, coskewness and cokurtosis separately. Following the definition in Bakshi et al. (2003) and Conrad et al. (2013), the market beta, coskewness and cokurtosis are defined as:

\[
BETA_{it} = \frac{\mathbb{E}[(r_{it} - \mathbb{E}[r_{it}]) (r_{mt} - \mathbb{E}[r_{mt})]}}{\text{Var}(r_{mt})}, \tag{1}
\]

\[
COSK_{it} = \frac{\mathbb{E}[(r_{it} - \mathbb{E}[r_{it}]) (r_{mt} - \mathbb{E}[r_{mt})]^2]}{\sqrt{\text{Var}(r_{it}) \text{Var}(r_{mt})}}, \tag{2}
\]

\[
COKT_{it} = \frac{\mathbb{E}[(r_{it} - \mathbb{E}[r_{it}]) (r_{mt} - \mathbb{E}[r_{mt})]^3]}{\text{Var}(r_{it}) \text{Var}(r_{mt})}. \tag{3}
\]

All stocks are sorted on the three characteristics above and divided into five groups based on the 20th, 40th, 60th and 80th percentiles. Then we form annually rebalanced portfolios, value weighted based on the capitalization of each stock. We denote by BETA1 (COSK1, COKT1) the portfolio formed by stocks with the highest beta (respectively, coskewness, cokurtosis), and BETA5 (COSK5, COKT5) denotes the portfolio formed by stocks with the lowest beta (coskewness, cokurtosis).

Turn now to the currency (FX) portfolios. We sort portfolios based on the total order flow. Menkhoff et al. (2013) point out that it is essential to standardize order flows before sorting, as the absolute size of order flows are quite different across currencies. Following the procedure in their paper, we standardize order flows by dividing by their standard deviation to allow for reasonable comparisons:

\[
\tilde{OF}_{it} = \frac{OF_{it}}{\sigma(OF_{it})} \tag{4}
\]
where \( \tilde{OF}_{it} \) denotes standardized order flow using a 52-week rolling window and \( OF_{it} \) denotes raw order flow. Further, \( OF_1 \) denotes the portfolio formed by currencies with the highest order flow and \( OF_5 \) denotes the portfolio formed by currencies with the lowest order flow.

### 2.2 Modeling the Marginal Density

To generate two series of observations, we fit an AR model to the conditional mean \( \mu_{it} \) of the returns of each time series

\[
r_{it} = c_i + \sum_{k=1}^{p} \phi_{i,k} r_{i,t-k} + \varepsilon_{it}, \quad i = 1, 2, \ldots \text{, where } \varepsilon_{it} = \sigma_{it} z_{it}
\]

and an asymmetric GARCH model, namely GJR-GARCH(1,1,1) (see Glosten et al., 1993), to the conditional variance

\[
\sigma_{it}^2 = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 + \gamma_i \varepsilon_{i,t-1}^2 I_{i,t-1}
\]

where \( I_{i,t-1} = 1 \) if \( \varepsilon_{i,t-1} < 0 \), and \( I_{i,t-1} = 0 \) if \( \varepsilon_{i,t-1} \geq 0 \). We allow each series to have time-varying conditional mean and variance, and we also assume that the standardized errors \( z_{it} = (r_{it} - \mu_{it}) / \sigma_{it} \) have an identical conditional distribution. Let \( z \) be a random variable with continuous distribution \( F \), then \( F(z) \sim U[0,1] \) (see McNeil et al., 2005). For the parametric model, we assume that \( z_{it} \) follow the skewed Student’s \( t \) distribution of Hansen (1994).

\[
z_{it} \sim F_{skew-t}(\hat{\eta}_i, \hat{\lambda}_i), \quad U_{it} = F_{skew-t}(z_{it}; \eta_i, \lambda_i)
\]

where \( \eta_i \) denotes the degrees of freedom, \( \lambda_i \) the skewness parameter, and \( U_{it} \) the probability integral transformation. Hence, we can easily compute the probability given the estimates of parameters; \( \hat{\mu}_{it}, \hat{\sigma}_{it}, \hat{\eta}_i \) and \( \hat{\lambda}_i \). For the nonparametric model, we use the empirical distribution function to obtain the estimate of \( F_i \):

\[
\hat{F}_i(z) \equiv \frac{1}{T+1} \sum_{t=1}^{T} \mathbb{1}\{\hat{z}_{it} \leq z\}, \quad \hat{U}_{it} = \hat{F}_i(\hat{z}_{it}).
\]

### 2.3 Computation of Asymmetric Dependence

A primary goal of our paper is to establish how the characteristic-sorted portfolio returns covary and whether their dependence structures are asymmetric. Consequently, we consider three different dependence structures: The threshold correlation; the quantile dependence; and the tail dependence.
Following Longin and Solnik (2001) and Ang and Chen (2002), the threshold correlation for probability level $p$ is given by

\[ \rho^- = \text{Corr}(r_{1t}, r_{2t} | r_{1t} \leq r_1(p) \text{ and } r_{2t} \leq r_2(p)) \text{ if } p \leq 0.5 \]  
\[ \rho^+ = \text{Corr}(r_{1t}, r_{2t} | r_{1t} > r_1(p) \text{ and } r_{2t} > r_2(p)) \text{ if } p > 0.5 \]  

(9) (10)

where $r(p)$ denotes the corresponding empirical percentile for asset returns $r_{1t}$ and $r_{2t}$. In words, we compute the correlation between two assets conditional on both of them being less (respectively, greater) than their $p$th percentile value when $p \leq 0.5$ (respectively, $p > 0.5$). To examine whether this asymmetry is statistically significant, we consider a model-free test proposed by Hong et al. (2007). If the null hypothesis that $\rho^+ = \rho^-$ can be rejected, then there exists a linear asymmetric correlation between $r_{1t}$ and $r_{2t}$.

In contrast to threshold correlation (a scalar measure), quantile dependence provides a more precise measure of dependence structure as it contains more detailed information. In addition, from a risk management perspective, tails are more important than the center. Following Patton (2012), the quantile dependence can be defined as

\[ \lambda^q = \begin{cases} 
\mathbb{P}\{U_{1t} \leq q | U_{2t} \leq q\} = \frac{C(q,q)}{q} & \text{if } 0 < q \leq 0.5 \\
\mathbb{P}\{U_{1t} > q | U_{2t} > q\} = \frac{1-2q+C(q,q)}{1-q} & \text{if } 0.5 < q \leq 1
\end{cases} \]  

(11)

and nonparametrically estimated by

\[ \hat{\lambda}^q = \begin{cases} 
\frac{1}{Tq} \sum_{t=1}^{T} 1\{\hat{U}_{1t} \leq q, \hat{U}_{2t} \leq q\} & \text{if } 0 < q \leq 0.5 \\
\frac{1}{T(1-q)} \sum_{t=1}^{T} 1\{\hat{U}_{1t} > q, \hat{U}_{2t} > q\} & \text{if } 0.5 < q < 1
\end{cases} \]  

(12)

where $C$ denotes the corresponding copula function (defined shortly in Equation (16)).

The tail dependence coefficient (TDC) is a measure of the degree of dependence in the tail of a bivariate distribution (see McNeil et al., 2005; Frahm et al., 2005; Joe et al., 2010, among others). Let $z_1$ and $z_2$ be random variables with continuous distribution functions $F_1$ and $F_2$. Then the coefficients of upper and
lower tail dependence of \( z_1 \) and \( z_2 \) are

\[
\lambda^L = \lim_{q \to 0^+} \frac{\mathbb{P}\{z_2 \leq F_2^{-1}(q), z_1 \leq F_1^{-1}(q)\}}{\mathbb{P}\{z_1 \leq F_1^{-1}(q)\}} = \lim_{q \to 0^+} \frac{C(q, q)}{q} \tag{13}
\]

\[
\lambda^U = \lim_{q \to 1^-} \frac{\mathbb{P}\{z_2 > F_2^{-1}(q), z_1 > F_1^{-1}(q)\}}{\mathbb{P}\{r_1 > F_1^{-1}(q)\}} = \lim_{q \to 1^-} \frac{1 - 2q + C(q, q)}{1 - q} \tag{14}
\]

The coefficients can be easily calculated when the copula \( C \) has a closed form. The copula \( C \) has upper tail dependence if \( \lambda^U \in (0, 1] \) and no upper tail dependence if \( \lambda^U = 0 \). A similar conclusion holds for the lower tail dependence. If the copulas are symmetric, then \( \lambda^L = \lambda^U \), otherwise, \( \lambda^L \neq \lambda^U \) (see Joe, 1997). McNeil et al. (2005) state that the copula of the bivariate \( t \) distribution is asymptotically dependent in both the upper and lower tail. The rotated Gumbel copula is an asymmetric Archimedean copula, exhibiting greater dependence in the negative tail than in the positive. Both of them allow heavier negative tail dependence than the Gaussian copula and are widely used in the finance literature. We use both the Student’s \( t \) copula and the rotated Gumbel copula to estimate the tail dependence coefficient between portfolios.

### 2.4 Copulas

In this section, we provide a brief introduction to copulas. The Sklar (1959) theorem allows us to decompose a conditional joint distribution into marginal distributions and a copula. It allows considerable flexibility in modeling the dependence structure of multivariate data. Let \( \mathbf{r} = (r_1, \ldots, r_d)' \), \( d \geq 2 \) be a \( d \)-dimensional random vector with joint distribution function \( F(r_1, \ldots, r_d) \) and marginal distribution functions \( F_i(r_i) \), \( i \in \{1, \ldots, d\} \). According to Sklar’s theorem, there exist a \( d \)-dimensional copula \( C : [0, 1]^d \to [0, 1] \) such that

\[
F(r_1, \ldots, r_d) = C(F_1(r_1), F_2(r_2), \ldots, F_d(r_d)) \quad \forall \mathbf{r} \in \mathbb{R}^d, \tag{15}
\]

and the copula \( C(u_1, \ldots, u_d) \), \( u_i \in (0, 1) \) is unique if the marginal distributions are continuous. Let \( F_i^{-1} \) denote the generalized inverse distribution function of \( F_i \), then \( F_i^{-1}(u_i) = r_i \). The copula \( C(u_1, \ldots, u_d) \) of a multivariate distribution \( F(r_1, \ldots, r_d) \) with marginals \( F_i(r_i) \), \( i \in \{1, \ldots, d\} \) is given by

\[
C(u_1, \ldots, u_d) = F(F_1^{-1}(u_1), F_2^{-1}(u_2), \ldots, F_d^{-1}(u_d)) \tag{16}
\]
If $F_i$ has density $f_i$, $i \in \{1, \ldots, d\}$ and copula $C$ has density $c$, then it satisfies

$$c(u_1, \ldots, u_d) = \frac{f(F_1^{-1}(u_1), F_2^{-1}(u_1), \ldots, F_d^{-1}(u_d))}{\prod_{i=1}^{d} f_i(F_i^{-1}(u_i))} = \left. \frac{\partial^n C(u_1, \ldots, u_d)}{\partial u_1 \cdots \partial u_d} \right|_{u_i=F_i^{-1}(u_i)}$$

(17)

Sklar’s theorem implies that for multivariate distribution functions, the univariate marginals and the dependence structure can be separated. In our study, we only consider the case of a bivariate copula.

### 2.5 Generalized Autoregressive Score (GAS) Model

We estimate the dynamic copula model based on the Generalized Autoregressive Score (GAS) model of Creal et al. (2013). We assume that the correlation parameter $\delta_t$ is dynamic and is updated as function of its own lagged value. To make sure that it always lies in a pre-determined range (e.g. $\delta_t \in (-1, 1)$), the GAS model utilizes a strictly increasing transformation. The transformed parameter is denoted by $g_t$:

$$g_t = h(\delta_t) \iff \delta_t = h^{-1}(g_t),$$

(18)

where $\delta_t = (1 - e^{-g_t}) / (1 + e^{-g_t})$. Further, the updated transformed parameter $g_{t+1}$ is a function of a constant $\bar{\omega}$, the lagged transformed parameter $g_t$, and the standardized score of the copula log-likelihood $Q_t^{-1/2} s_t$:

$$g_{t+1} = \bar{\omega} + \eta Q_t^{-1/2} s_t + \varphi g_t,$$

(19)

where

$$s_t \equiv \frac{\partial \log c(U_1, U_2; \delta_t)}{\partial \delta_t} \text{ and } Q_t \equiv E_{t-1} \left[ s_t s_t' \right].$$

Since the GAS model is an observation driven model, the parameters can be estimated by using maximum likelihood estimation

$$\hat{\delta}_t = \arg\max_{\delta} \sum_{i=1}^{n} \log c(U_{1t}, U_{2t}; \delta_t).$$

(20)

The dynamic copulas are parametrically estimated using maximum likelihood estimation. When the marginal distributions are estimated using the skewed Student’s $t$ distribution, the resulting joint distribution is fully parametric. When the marginal distribution is estimated by the empirical distribution function, then the resulting joint distribution is semiparametric. More details can be found in the appendix.
2.6 Value-at-Risk Forecasts

We now turn to VaR forecasts. To simplify our analysis, we restrict attention to the bivariate case and we arbitrarily consider the weights on the two portfolios to be equal and opposite (long one unit and short one unit). Then our long-short portfolio return $r_{pt}$ is approximately equal to the following:

$$r_{pt} = r_{ht} - r_{lt},$$

where $r_{ht}$ (respectively, $r_{lt}$) is the ex post return of the portfolio sorted on the highest (respectively, lowest) beta, coskewness, cokurtosis or order flow.

The ex ante VaR of the long-short portfolio at time $t$ and confidence level $\alpha \in (0, 1)$, is defined as:

$$\text{VaR}_{pt}(\alpha) = \inf \left\{ x \mid P(r_{pt} \leq x | F_{t-1}) \leq \alpha \right\},$$

where $F_{t-1}$ represents the information available at $t - 1$. In our study, $\alpha$ is assumed to be either 0.05 or 0.01, and we report results focusing on 0.01 (99% confidence level) which is the most widely used value for market risk management. Once the dynamic copula parameters have been estimated, Monte Carlo simulation is used to generate 5000 values of $r_{ht}^{(s)}$ and $r_{lt}^{(s)}$ and, hence, of $r_{pt}^{(s)}$. From the empirical distribution of $r_{pt}^{(s)}$, the desired quantile VaR are estimated.

3 Data and Marginal Distribution Modeling

3.1 Description of Data

First, data on stock prices are obtained from Datastream. Daily returns of 500 stocks listed in the S&P 500 and 100 stocks listed in FTSE 100 are used to construct thirty different portfolios - fifteen for US equities and fifteen for the UK. The fifteen consist of one for each of the three characteristics (ex ante beta, coskewness and cokurtosis), divided into five groups based on the 20th, 40th, 60th and 80th percentiles. Our data, spanning the period of the global financial crisis of 2007-2009 and European sovereign debt crisis of 2010-2011, go from January 4, 2000 to December 31, 2012, resulting in 3,268 daily observations for each stock in US and 3,283 daily observations for each stock in UK. Second, data on FX are obtained from UBS. It consists of weekly order flows for up to 10 currency pairs from November 2, 2001 to March
23, 2012, resulting in 543 weekly observations for each currency. Currencies are sorted into portfolios based on their total order flows. The definitions of HML portfolios are presented in Table 1.

The reasons that we investigate ex ante beta, coskewness, cokurtosis and order flow sorted portfolios are as follows: First, according to the CAPM, a stock’s expected excess return is proportional to its market beta; Second, the coskewness of a stock return with the market captures asymmetry in risk, especially downside risk, which is important for practitioners when computing VaR; Third, empirical work provides evidence that coskewness and cokurtosis can explain the cross-sectional variation of asset returns (see Harvey and Siddique, 2000; Dittmar, 2002; Conrad et al., 2013); Fourth, recent studies find that order flows are highly informative about future exchange rates (see Menkhoff et al., 2013).

Summary statistics for the high and low portfolio returns are presented in Panel A of Table 2. We find that the portfolio constructed from high beta stocks (i.e. BETA5) tends to offer relatively lower average returns than the portfolio constructed from low beta stocks (i.e. BETA1). Given the fact that the US and UK equity markets have fallen 2.02% and 16.13% from 2000 to 2012, this finding is not very surprising. The portfolios with negative average returns show relatively higher volatility than the portfolios with positive average returns in both stock markets and FX markets. These asymmetric responses to underlying market conditions are consistent with many previous studies. The skewness of the portfolio returns are non-zero while the kurtosis of the portfolio returns are significantly higher than 3 indicating that the empirical distributions of returns display heavier tails than a Gaussian distribution. Using the Ljung-Box Q-test, the null hypothesis of no autocorrelation is rejected at lag 5 and lag 10 except for OF5. The ARCH test of Engle (1982) indicates the significance of ARCH effects in all the series. We also find similar results for the HML portfolios in Panel B of Table 2. Overall, the summary statistics show the nonnormality, asymmetry, autocorrelation and heteroscedasticity of portfolio returns.

Figure 1 displays the scatter plots of the high and low portfolio pairs; (BETA1, BETA5), (COSK1, COSK5), (COKT1, COKT5) and (OF1, OF5). Further it provides threshold correlation coefficients at the center and at both the upper and lower tails of the empirical distribution. Beta portfolios have larger correlations at both tails than the correlations at the center in both stock markets. Coskewness portfolios
of the US stock market have smaller correlation at the center than the correlation at the lower tail while those of the U.K. stock market have larger correlations at both tails than the correlation at the center. Cokurtosis portfolios show similar patterns to coskewness portfolios. Unlike stock portfolios, order flow portfolios have a larger correlation at the center than the correlations at either tail. The common feature is that the lower tail correlation is larger than the upper tail correlation. This stylized fact is consistent with previous research. Overall, the scatter plots and the threshold correlation coefficients clearly show that the correlations between the respective high and low portfolios are nonlinear and asymmetric.

Before modeling the joint distribution of portfolio returns, it is necessary to select a suitable model for the marginal return distribution, because misspecification of the univariate model can lead to biased copula parameter estimates. To allow for autocorrelation, heteroskedasticity and asymmetry, we use the models introduced in Section 2 in Eq. (5) to (8).

First, we use the Bayesian Information Criterion (BIC) to select the optimal order of the AR model for the conditional mean up to order 5. Second, to allow for the heteroskedasticity of each series, we consider a group of GARCH models as candidates and find that the asymmetric GARCH model of Eq. (6) is preferred to the others based on their likelihood values. Thus, we consider the GJR-GARCH class of up to order (2,2,2) and select the optimal order by using BIC again. The model parameters are estimated by using maximum-likelihood estimation (MLE) and the results of AR and GARCH estimations are presented in Table 3. For each series, the variance persistence implied by the model is close to 1. For most of the series, the leverage effect parameters $\gamma$ are significantly positive implying that a negative return on the series increases volatility more than a positive return with the same magnitude. For the FX portfolios, sorted on order flow, we simply apply a GARCH(1,1) model as it provides higher likelihood values and smaller BIC.

The obvious skewness and high kurtosis of returns leads us to consider the skewed Student’s $t$ distribution of Hansen (1994) for residual modeling. We report the estimation results in Table 3. To evaluate the goodness-of-fit for the skewed Student’s $t$ distribution, the Kolmogorov-Smirnov (KS) and Cramer-von
Mises (CvM) tests are implemented and the $p$-values from these two tests are reported in Table 3. Our results suggest that the skewed Student’s $t$ distribution is suitable for residual modeling. Thus, in general, the diagnostics provides evidences that our marginal distribution models are well-specified and therefore, we can reliably use the combination of AR, GARCH and skewed Student’s $t$ distribution, allied to copulas to model the dependence structure.

4 Dependence: Dynamics and Asymmetry

This section seeks to accomplish two tasks. First, we describe the dynamic evolution of dependence between the high beta (respectively, coskewness, cokurtosis, order flow) portfolios and the low beta (coskewness, cokurtosis, order flow) portfolios, and examine whether it is statistically time-varying. If the variation of dependence between factor returns were not to be statistically significant, then there would be no reason to implement a dynamic model (due to its increased computational complexity). In addition, we wish to test whether the dependence structure has dramatically changed after the start of the global financial crisis of 2007-2009 and after the start of the European sovereign debt crisis of 2010-2011. Second, we measure asymmetric dependence using threshold correlation, copula-based quantile dependence and tail dependence and we test whether this asymmetry is significant.

4.1 Time-varying Dependence

There is considerable evidence that the conditional mean and conditional volatility of financial time series are time-varying. This, possibly, suggests the reasonable inference that the conditional dependence structure may also change through time. To visualize this variation, Figure 2 depicts time series plots of rolling 250-day (60-week for order flow portfolios) rank correlation between the high and low portfolios with 90% pointwise bootstrap confidence interval. However, the standard errors estimated are correct only under the null hypothesis that this correlation is not changing. The rolling rank correlations for all the equity portfolios increase significantly during 2000-2002, which is probably caused by the early 2000s economic recession that affected the European Union during 2000 and 2001 and the United States in 2002 and 2003, and the bursting of the dot com bubble. In general, all the rolling window rank correlations between the high and low portfolios increase from 2000 to 2012.

2The $p$-values are obtained based on the algorithm suggested in Patton (2012)
We now consider three tests for time-varying dependence. The first one is a naïve test for a break in rank correlation at specified points in the sample, see Patton (2006). A noticeable limitation of this test is that the break point of dependence structure (e.g. a specified date) must be known \textit{a priori}. The second test for time-varying dependence allows for a break in the rank correlation coefficient at some prior unspecified date, see Andrews (1993). The third test is the ARCH LM test for time-varying volatility, see Engle (1982). The critical values for the test statistic can be obtained by using a \textit{iid} bootstrap algorithm, see Patton (2012). The results of the above tests for time-varying dependence are summarized in Table 4.

Suppose there is no \textit{a priori} date for the timing of a break, we first consider naïve tests for a break at three chosen points in our sample, at $t^*/T \in \{0.15, 0.50, 0.85\}$, which corresponds to the dates 10-Dec-2001, 03-Jul-2006, and 17-Jan-2011. Then we consider another test in Andrews (1993) for a dependence break of unknown timing. As can be seen from Table 4, for almost all the equity portfolios, the $p$-value is significant at the 5% significance level showing clear evidence against a constant rank correlation with a one-time break. To detect whether the dependence structures between the high and low portfolios significantly changed during the global financial crisis of 2007-2009 and the European sovereign debt crisis of 2010-2011, we use 15-Sep-2008 (the collapse of Lehman Brothers) and 01-Jan-2010 (EU sovereign debt crisis) as two break points. We find that the dependence between BETA1 and BETA5 significantly changed around those dates, as all the $p$-values are fairly small. For other portfolio pairs, time homogeneity of the dependence structure is rejected by at least one test.

Overall, we find evidence against time homogeneity of the dependence structure between the standardized residuals of portfolios. This result shows that the standard portfolio diversification and risk management techniques based on constant correlations (or dependence) are inadequate, especially during financial crises. Thus, the heterogeneity of dependence provides us a strong motivation to introduce a dynamic copula model for financial forecasting.

4.2 \textit{Asymmetric Dependence}

Standard models fail to take into account a noteworthy feature during financial crises that asset returns often become more highly correlated (in magnitude). To test for the presence of this feature, we use
threshold correlations, Eq. (16). Figure 3 shows the lower and upper threshold correlations for the high portfolio versus low portfolio. The lower threshold correlations are always greater than the upper threshold correlation indicating that portfolios are more correlated when both of them perform poorly. From a portfolio management perspective, this feature is extremely important. For instance, the correlation between OF1 and OF5 is relatively low suggesting that diversification is high, but when both OF1 and OF5 have poor performances, their correlation can go up to more than 0.55. Therefore, the bivariate normal distribution cannot well describe the “true” dependence for the following reasons: First, the normal distribution is symmetric. Second, in the bivariate normal distribution, the threshold correlation approaches 0 when the threshold is asymptotically close to 0 or 1. To find out whether this asymmetry is statistically significant, we perform the symmetry tests of Hong et al. (2007). Table 5 reports the test results and shows that, as measured by threshold correlation, over half of the portfolios are significantly asymmetric: \(HML(Beta, US/UK), HML(Cokt, UK)\) and \(HML(OF)\).

Although threshold correlation offers some insights, it is still based on (linear) correlation and, therefore, does not take into account nonlinear information. To capture nonlinear dependence, we consider copula-based quantile dependence and tail dependence. Compared with (linear) correlation, the key advantage of copulas is that they are a “pure measure” of dependence, which cannot be affected by the marginal distributions (see Nelsen, 2006).

Quantile dependence measures the probability of two variables both lying above or below a given quantile (e.g. upper or lower tail) of their univariate distributions. Examining different quantiles allows us to focus on different aspects of the relationship. In Figure 4, we present the quantile dependence between the high beta (coskewness, cokurtosis, order flow) portfolios and the low beta (coskewness, cokurtosis, order flow) portfolios as well as the difference in upper and lower quantile dependence. For every portfolio pair, the left panel of Figure 4 shows the estimated quantile dependence plot, for \(q \in [0.025, 0.975]\), along with 90% (pointwise) i.i.d. bootstrap confidence intervals, and the right panel shows the difference between the upper and lower portions of this plot, along with a pointwise confidence interval for this difference. As expected, the confidence intervals are narrower in the middle of the distribution (values of \(q\) close to 1/2) and wider near the tails (values of \(q\) near 0 or 1). Figure 4 also shows that observations in the lower tail are slightly more dependent than observations in the upper tail, with the difference
between corresponding quantile dependence probabilities being as high as 0.3. The confidence intervals show that these differences are borderline significant at the 10% significance level, with the upper bound of the confidence interval on the difference lying around zero for most values of $q$. From the perspective of risk management, the dynamics implied by our empirical results may be of particular importance in the lower tails, because of its relevance for the portfolio VaR. We present the dynamic evolution of tail dependence coefficient (TDC) between the standardized residuals of the high and low portfolios in Figure 5. The dependence between equity portfolios, such as BETA1 and BETA5, is quite low in 2003 and has significantly increased since then. In the US equity market, the lower tail dependence (LTD) is relatively close to or even lower than the upper tail dependence before the global financial crisis of 2007-2009. However, the LTD has become greater than UTD following 2007. In the UK market, the LTD is always greater than the UTD. This phenomenon can be interpreted from behavioural finance theory that investors dislike losses more than they like gains of the same magnitude. For the FX portfolios, things are quite different. The LTD substantially decreased from 2006 to 2010 and increased from 2011 to 2012, while the UTD is relatively stable. Therefore there is a significant difference between tails before the global financial crisis of 2007-2009 but the difference disappears since the crisis.

[ INSERT FIGURE 4, 5 AND TABLE 6 ABOUT HERE ]

Next, we consider the tail dependence, which is a copula-based measure of dependence between extreme events. We employ the rotated Gumbel copula and the Student’s $t$ copula to estimate the tail dependence coefficients. All the coefficients are estimated by both parametric and semiparametric copula methods (detailed in the appendix). To avoid possible model misspecification, we use the nonparametric estimation method proposed by Frahm et al. (2005) as a robustness check and the results are consistent with results generated by the parametric and semiparametric methods. Table 6 reports the coefficients of lower tail dependence (LTD) and upper tail dependence (UTD) and the difference between them. The coefficients are estimated using McNeil et al. (2005). For example, the lower tail coefficient estimated by rotated Gumbel copula (respectively, Student’s $t$ copula) for BETA1 and BETA5 in the US equity market is 0.256 (respectively, 0.171) and the upper tail coefficient estimated by rotated Gumbel copula (Student’s $t$ copula) is 0.099 (respectively, 0.018). Then we find the significant difference between the upper and lower tail dependence coefficients. In the UK equity market, we also find evidence of asymmetric dependence in that all the portfolio pairs exhibit greater correlation during market downturns than market
upturns. This finding about asymmetric dependence between the high beta (coskewness, cokurtosis) portfolio and the low beta (coskewness, cokurtosis) portfolios is new. It is possibly associated with the fact that investors have more uncertainty about the economy, and therefore pessimism and panic spread from one place to another more quickly during market downturns. Another possible explanation is the impact of liquidity risk. Some “uncorrelated” liquid assets suddenly become illiquid during market downturns, and, therefore, even a small trading volume can lead to huge co-movements. The semiparametric tail dependence approach (that is nonparametric approach for the marginal distributions and and parametric for the copula estimation) and the nonparametric tail dependence approach of Frahm et al. (2005) are used as robustness checks and both of them provide similar results to the parametric approach. Therefore, in the equity markets, we can reject the null hypothesis of symmetric dependence and conclude that for, most portfolio pairs, dependence is significantly asymmetric. In contrast, the results for the FX portfolios support symmetric dependence.

5 Measuring Portfolio Risk with Dynamic Copulas

In this section, we evaluate the economic significance of the dynamic copula model by forecasting our portfolio VaR. We consider 12 copulas including Normal, Student’s $t$, generalized hyperbolic skew Student’s $t^3$, Clayton, Rotated Clayton, Clayton mixture, Gumbel, Rotated Gumbel, Gumbel mixture, Plackett, Frank and Symmetrized Joe-Clayton, as candidates to model the dependence between BETA1 (COSK1, COKT1, OF1) and BETA5 (COSK5, COKT5, OF5). Using goodness-of-fit tests (see Rémillard, 2010), we find that the Student’s $t$ copula and the generalized hyperbolic skew Student’s $t$ copula give the best fit over the in-sample period in most cases. Thus, we employ them to model the dependence and forecast our portfolio VaR.

In order to forecast one-step ahead VaR, we use a rolling window instead of the full sample period and the rolling window size is set at 250 (one trading year for equity data and 5 trading years for FX data) for all the data sets. All the models are recursively reestimated throughout the out-of-sample period and the correlation coefficients of copulas are forecasted by the GAS model. We use the rolling windows to evaluate the out-of-sample forecasting performance. The backtesting evaluates the coverage ability and

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3 More details about GH skew Student’s $t$ copula can be found in Demarta and McNeil (2005) and Christoffersen et al. (2012)

4 These results are available upon request.
the statistical accuracy of the VaR models. The coverage ability is evaluated by the empirical coverage
probability (hereafter ECP) and Basel Penalty Zone (hereafter BPZ). The statistical accuracy is evaluated
by the conditional coverage test (hereafter CC test; Christoffersen, 1998) and the dynamic quantile test
(hereafter DQ test; Engle and Manganelli, 2004).

We first define the failure of the VaR model as the event that a realized return is not covered by the
predicted VaR. We identify it by the indicator function taking the value unity in the case of failure:

\[ I_s = 1 \left\{ r_s < \hat{VaR}_s(\alpha|\mathcal{F}_{s-1}) \right\}, \quad s = 1, \ldots, N, \]  

(23)

where \( \hat{VaR}_s(\alpha|\mathcal{F}_{s-1}) \) is the VaR forecast based on the information set at \( s - 1 \), denoted by \( \mathcal{F}_{s-1} \), with a
nominal coverage probability \( \alpha \). Henceforth, we abbreviate the notation \( \hat{VaR}_s(\alpha|\mathcal{F}_{s-1}) \)
to \( \hat{VaR}_s(\alpha) \).

ECP is calculated by the sample average of \( I_s \), \( \hat{\alpha} = \frac{1}{N-1} \sum_{s=1}^{N} I_s \) which is a consistent estimator of
the coverage probability. The VaR model for which ECP is closest to its nominal coverage probability
is preferred. BPZ is suggested by Basel Committee on Banking and Supervision (1996). It describes
the strength of the VaR model through the test of failure rate. It records the number of failures of the
99 percent VaR in the previous 250 business days. One may expect, on average, 2.5 failures out of the
previous 250 VaR forecasts given the correct forecasting model. The Basel Committee rules that up to
four failures are acceptable for banks and defines the range as a “Green” zone. If the failures are five or
more, the banks fall into a “Yellow” (5–9) or “Red” (10+) zone. The VaR model of which BPZ is “Green”
zezone is preferred.

Accurate VaR forecasts should satisfy the condition that the conditional expectation of the failure is
the nominal coverage probability:

\[ \mathbb{E}[I_s|\mathcal{F}_{s-1}] = \alpha. \]

(24)

Christoffersen (1998) shows that it is equivalent to testing if \( I_s|\mathcal{F}_{s-1} \) follows an i.i.d. Bernoulli distribution
with parameter \( \alpha \):

\[ H_0 : I_s|\mathcal{F}_{s-1} \sim i.i.d. Bernoulli(\alpha). \]

(25)

The CC test uses the LR statistic which follows the chi-squared distribution with two degrees-of-freedom
under the null hypothesis, Eq. (25). The DQ test is a general extension of the CC test allowing for more
time-dependent information of \( \{I_s\}_{s=1}^N \). The out-of-sample DQ test is given by

\[
DQ = \frac{(\mathbf{I}' \mathbf{Z})(\mathbf{Z}' \mathbf{Z})^{-1}(\mathbf{Z}' \mathbf{1})}{\alpha (1 - \alpha)} \sim \chi^2_{p+2},
\]

where \( \mathbf{I} = (\bar{I}_{p+1}, \bar{I}_{p+2}, \ldots, \bar{I}_N)' \), \( \bar{I}_s = I_s - \alpha \), \( \mathbf{Z} = (z_{p+1}, \ldots, z_N)' \) and \( \mathbf{z}_s = (1, \bar{I}_{s-1}, \ldots, \bar{I}_{s-p}, \hat{VaR}_s (\alpha))' \). We use the first four lags for our evaluation, i.e., \( \mathbf{z}_s = (1, \bar{I}_{s-1}, \ldots, \bar{I}_{s-4}, \hat{VaR}_s (\alpha))' \).

For the UK portfolios, we estimate the VaR models using 250 business days over the period 3 Jan. 2000 - 15 Dec. 2000, and compute the one-day-ahead forecast of the 99 percent VaR for 18 Dec. 2000. We conduct rolling forecasting by moving forward a day at a time and end with the forecast for 31 Dec. 2012. This generates 3,033 out-of-sample daily forecasts. Next we repeat the same process for the US portfolios. It starts with the forecast for 18 Dec. 2000 and ends with the forecast for 31 Dec. 2012. This generates 3,018 out-of-sample daily forecasts. Finally, we repeat the same process for the FX portfolios. It starts with the forecast for 18 Aug. 2006 and ends with the forecast for 23 Mar. 2012. This generates 295 out-of-sample weekly forecasts.

### 5.1 Coverage Ability

We evaluate the coverage ability by ECP and BPZ as follows: First, we calculate ECP for each portfolio and then report bias and Root Mean Square Error (RMSE). Bias is the average deviation of ECP from the nominal coverage probability (1% in our case). The smaller the bias is, the more accurate the VaR forecast is. RMSE is the average of the squared deviation. It shows the dispersion of ECP from the nominal coverage probability. It makes up for the defect of bias due to the offset of positive and negative deviations. Financial regulators would prefer a VaR model with, simultaneously, a small bias and small RMSE. Second, BPZ describes the coverage ability of the VaR model through the test of failure rate. It counts the number of failure over the previous 250 business days.

#### 5.1.1 Empirical Coverage Probability

Table 7 presents the ECPs of the VaR models. First, the bias of the parametric time-varying skew \( t \) copula models is 0.00% and the bias of the semiparametric dynamic skew \( t \) copula models is 0.001 which are much smaller than those of the other models. It shows that the ECPs of the dynamic copula models are
very close to the nominal one. In addition, their RMSEs are significantly smaller than the others. The bias of the static t copula is 0.21% which is much greater than those of the dynamic t copula models, and the RMSE is about four times the RMSE of the dynamic t copula. This is clear evidence of the superiority of the dynamic copula model.

Second, the univariate models show a large positive bias. This implies that their VaRs are under-forecasted. The RMSEs are also very large - between four times (historical simulation, hereafter HS, 0.64%) and fifteen times (CAViaR\(^5\), 2.52%) greater than the dynamic t and skew t copula models. We infer that the poor results are due to their inherent limitations. Since historical simulation (HS) assumes a time-invariant asset return distribution, forecasting could be inaccurate in the presence of regime (mean or volatility) shifting (Barone-Adesi, et al., 2002). The parametric model is likely to suffer from two defects: First, there could be a selection bias when we choose the wrong distribution of asset returns. Second, there could be a misspecification error when we specify the dynamics of key parameters. In addition, it is hard to get reasonable results with a simple quantile specification for CAViaR.

Finally, the bias of the multivariate GARCH models range from 0.20% to 0.24%. These are greater than for the dynamic copula models but significantly smaller than for the univariate models. Their RMSEs are also much greater than those of the dynamic copula models. They are however, in general, smaller than those of the univariate models. It shows that multivariate GARCH modeling of risk is more advantageous than univariate modeling in the presence of dependence among different risks. However, it also shows that copula modeling of the tail dependence is more accurate than multivariate GARCH modeling of the central second moment (covariance).

[ INSERT TABLE 7 ABOUT HERE ]

5.1.2 Basel Penalty Zone

Table 8 presents the BPZ of the VaR models. We consistently find that the dynamic t copula models show the best performance. First, the dynamic copula models achieve the Green zone for all the portfolios. The static copula models also achieves the Green zone for 12 portfolios. Second, the univariate models are warned by the Yellow zone for at least 4 portfolios (FEVT). CAViaR gets 13 Yellow warnings. Especially, CAViaR denotes the Conditional Autoregressive Value-at-Risk by Regression Quantiles. We use it as one of the benchmark models. More details can be found in Engle and Manganelli (2004).
the Red zone is imposed on GJR and EGARCH for the FX portfolio. Third, the multivariate GARCH models achieve the Green zone for more than 10 portfolios.

The BPZ evaluation shows that multivariate models have better coverage ability than univariate models. Copula modeling of tail dependence is more accurate in forecasting extreme events than multivariate GARCH modeling of the centred second moment (covariance). However, we must keep in mind that only the dynamic copula improves the forecasting accuracy.

[ INSERT TABLE 8 ABOUT HERE ]

5.2 Statistical Accuracy

We evaluate the statistical accuracy by the CC test and the DQ tests as follows: We calculate both statistics for each portfolio and test them at the 5% significance level. Then we report the number of rejected portfolios.

5.2.1 Conditional Coverage Test

Panel A of Table 9 reports the CC test results. First, the dynamic $t$ copula models are rejected for 2 (parametric) and 1 portfolios (semiparametric) whilst the static $t$ copula is rejected for 6 portfolios. Also, the dynamic skew $t$ copula models are rejected for 2 (parametric) and 2 portfolios (semiparametric) whilst the static skew $t$ copula is rejected for 5 portfolios. This shows that the dynamic copula more accurately forecasts extreme events than the static copula. Second, the univariate models are rejected for at least 6 portfolios (FEVT) and at most 12 portfolios (CAViaR). The poor test results could be explained by their poor coverage ability. Third, multivariate GARCH models are also rejected for more than 7 portfolios, which could be related to the large RMSE of their ECPs. Their average ECPs are close to the nominal coverage probability but individual ECPs show considerable variability across the portfolios. Hence, many individual portfolios are rejected. Therefore, the results of the CC test show that the dynamic copula models can forecast extreme events most accurately.

[ INSERT TABLE 9 ABOUT HERE ]
5.2.2 Dynamic Quantile Test

Panel B of Table 10 reports the results of the DQ test. Although the number of rejections increases, the results are qualitatively consistent with those of the CC test. Firstly, the dynamic $t$ copula models are rejected for 4 (parametric) and 6 portfolios (semiparametric), respectively. The static $t$ copula is rejected for 8 portfolios. Also, the dynamic skew $t$ copula models are rejected for 2 (parametric) and 4 portfolios (semiparametric), respectively. The static skew $t$ copula is rejected for 7 portfolios. It is qualitatively consistent with the CC test that the dynamic copula more accurately forecast extreme events than the static copula. The skew $t$ copula outperforms $t$ copula in DQ test as the skewed version of copula takes into account asymmetric dependence between assets. Secondly, the univariate models are rejected for more than 11 portfolios (except for FEVT (7 portfolios)). This suggests that the forecasting accuracy of the univariate models is too poor to be used, by banks, for internal VaR models. Finally, the multivariate GARCH models show the same number of rejections as that of the CC test except for DCC. BEKK and CCC are rejected for 7 and 8 portfolios which are smaller numbers of rejections than for the univariate models. The rejection numbers are however more than those of the dynamic copula models. Therefore, the DQ test also shows that the dynamic copula has statistically the most accurate forecasting ability.

\[ \text{[ INSERT TABLE 10 ABOUT HERE ]} \]

In summary, firstly, the multivariate models show better coverage ability and statistical accuracy than the univariate models. Secondly, the copula models more accurately forecast extreme events than the multivariate GARCH models. Thirdly, a dynamic copula is much more effective than a static copula in forecasting an extreme event. Finally, the dynamic GH skew $t$ copula, which takes into account asymmetric dependence, generally performs better than the dynamic $t$ copula. This result is of particular importance as it provides new evidence that the skew $t$ copula can improve the precision of forecasting.

6 Conclusion

This paper empirically addresses three related questions to improve our understanding of the dependence structure between financial assets with different characteristics under various market conditions and shows the economic significance of dynamic copula-based models from a risk management perspective. Our findings are novel as we go beyond the earlier copula literature that investigates the dependence across single
assets and explore dependence in a cross-sectional setting by forming characteristics-based portfolios of stocks and currencies. We sort stocks listed on the S&P 500 and the FTSE 100 into portfolios based on their ex ante beta, coskewness and co-kurtosis, and we also sort ten major currencies into five portfolios based on their aggregated order flows.

First, we provide empirical evidence that the dependence between characteristic-sorted portfolios is significantly time-varying. Using empirical data, spanning recent financial crises, we conclude that the returns of portfolios exhibit time-varying dependence and that the dependence has increased in recent years. Therefore, it provides strong support and motivation to apply dynamic copulas in dependence modeling.

Second, we use several tests to verify the presence of asymmetric dependence between high beta (coskewness, cokurtosis, order flow) portfolios and low beta (coskewness, cokurtosis, order flow) portfolios, in both the equity and FX markets. Our empirical results confirm this asymmetry and show that most portfolio pairs have stronger dependence during market downturns than during market upturns. Our conclusion strongly confirms the results in the extant literature, see Longin and Solnik (2001), Ang and Chen (2002), Patton (2006) and many others. It has wide implications for empirical asset pricing and asset allocation as well as for risk management.

Third, we apply a dynamic copula framework based on Creal et al. (2013) to predict portfolio VaR. This dynamic copula model has several attractive properties for VaR forecasting. The most attractive one is that it not only takes into account common features of univariate distributions, such as heteroscedasticity, skewness, fat tails, but also captures asymmetric and time-varying dependency between time series. All the models are estimated either parametrically, with the marginal distributions and the copula specified as belonging to parametric families, or semiparametrically, where the marginal distributions are estimated nonparametrically. In order to select the most appropriate models, goodness-of-fit tests are utilized. Several widely used univariate and multivariate VaR models are also considered for comparison. Backtestings are included in the evaluation process as well. Overall, our study provides new evidence that the GAS-based dynamic asymmetric copula model can offer more accurate VaR forecasts.

Taken together, these empirical findings indicate the statistical and economic significance of incorporating asymmetric and time-varying dependence in risk management. They can help investors better understand the co-movement between the portfolios with different characteristics, and control portfolio risk more effectively under different market conditions. Moreover, we empirically prove that the dynamic
copula-based model can provide financial institutions, such as banks, with a more powerful and precise model to forecast market risk and adjust minimum capital requirements.

References


Appendix

Estimation of Parametric Copula Model

The log-likelihood of a fully parametric copula model for conditional distribution of $r_t$ takes the form:

\[ L(\theta) = \prod_{t=1}^{T} f(r_t|F_{t-1}; \theta) \]

\[ = \prod_{t=1}^{T} \left[ c_t(u_1, \ldots, u_d|F_{t-1}; \theta_C) \prod_{i=1}^{N} f_{i,t}(r_{i,t}|F_{t-1}; \theta_i) \right] \]

with log-likelihood

\[ \sum_{t=1}^{T} \log f(r_t|F_{t-1}; \theta) = \sum_{t=1}^{T} \sum_{i=1}^{d} \log f_{i,t}(r_{i,t}|F_{t-1}; \theta_i) \]

\[ + \sum_{t=1}^{T} \log c_t(F_{1,t}(r_{1,t}|F_{t-1}; \theta_1), \ldots, F_{d,t}(r_{d,t}|F_{t-1}; \theta_d)|F_{t-1}; \theta_C) \]

where $\theta$ denotes the parameter vector for the full model parameters, $\theta_i$ denotes the parameters for the $i$th marginals, $\theta_C$ denotes the parameters of copula model and $F_{t-1}$ denotes the information set at time $t - 1$. Following the two-stage maximum likelihood estimation (also known as the Inference method for marginals) of Joe and Xu (1996), we first estimate the parameters of marginal models using maximum likelihood:

\[ \hat{\theta}_i = \arg\max_{\theta_i} \sum_{t=1}^{T} \log f_{i,t}(r_{i,t}|F_{t-1}; \theta_i), \; i = 1, \ldots, N \]  

(29)

and then using the estimations in the first stage, we calculate $F_{i,t}$ and estimate the copula parameters via maximum likelihood:

\[ \hat{\theta}_C = \arg\max_{\theta_C} \sum_{t=1}^{T} \log c_t(F_{1,t}(r_{1,t}|F_{t-1}; \theta_1), \ldots, F_{d,t}(r_{d,t}|F_{t-1}; \theta_d)|F_{t-1}; \theta_C) \]  

(30)

Estimation of Semiparametric Copula Model

In the semiparametric estimation (also known as Canonical Maximum Likelihood Estimation), the univariate marginals are estimated nonparametrically using the empirical distribution function and the copula
model is again parametrically estimated via maximum likelihood.

\[ \hat{F}_i(z) \equiv \frac{1}{T+1} \sum_{t=1}^{T} 1\{\hat{z}_{i,t} \leq z\} \]  

(31)

\[ \hat{u}_{i,t} \equiv \hat{F}_i(z) \sim Unif(0,1), \; i = 1, 2, ..., N \]  

(32)

\[ \hat{\theta}_C = \arg\max_{\theta_C} \sum_{t=1}^{T} \log c_t(\hat{u}_{1,t}, ..., \hat{u}_{i,t}|\mathcal{F}_{t-1}; \theta_C) \]  

(33)

where \( z_{i,t} \) are the standardized residuals of the marginal model and \( \hat{F}_i \) is different from the standard empirical CDF by the scalar \( \frac{1}{n+1} \) (in order to ensure that the transformed data cannot be on the boundary of the unit interval \([0, 1]\)).
Figure 1: The Scatter Plots for Portfolio 1 (High) and Portfolio 5 (Low)

Panel A. US Stock Market

\[ \rho_L = 0.63, \, \rho_C = 0.42, \, \rho_U = 0.52 \]

**Beta Portfolio**

\[ \rho_L = 0.70, \, \rho_C = 0.63, \, \rho_U = 0.46 \]

**Cokurtosis Portfolio**

\[ \rho_L = 0.65, \, \rho_C = 0.61, \, \rho_U = 0.41 \]

**Cokurtosis Portfolio**

Panel B. UK Stock Market

\[ \rho_L = 0.61, \, \rho_C = 0.29, \, \rho_U = 0.30 \]

**Beta Portfolio**

\[ \rho_L = 0.61, \, \rho_C = 0.46, \, \rho_U = 0.53 \]

**Cokurtosis Portfolio**

\[ \rho_L = 0.67, \, \rho_C = 0.47, \, \rho_U = 0.68 \]

**Cokurtosis Portfolio**

Panel C. FX Market

\[ \rho_L = 0.36, \, \rho_C = 0.42, \, \rho_U = 0.15 \]

**Order Flow Portfolio**

Note: This figure shows the scatter plots for different portfolio pairs, including \((BETA1, BETA5), (COSK1, COSK5), (COKT1, COKT5), (OF1, OF5)\). Three threshold correlation coefficients are used to demonstrate the asymmetric dependence between the portfolios:

\[ \rho_L = \text{Corr} \left( r_1, r_5 | r_1 \leq F_1^{-1}(0.15), r_5 \leq F_5^{-1}(0.15) \right), \]

\[ \rho_U = \text{Corr} \left( r_1, r_5 | F_1^{-1}(0.85) < r_1, F_5^{-1}(0.85) < r_5 \right), \]

\[ \rho_C = \text{Corr} \left( r_1, r_5 | F_1^{-1}(0.15) < r_1 \leq F_1^{-1}(0.85), r_5 \leq F_5^{-1}(0.15) \right), \]

where \(\rho_L, \, \rho_U\) and \(\rho_C\) denote the correlation coefficients at the lower tail, upper tail and center, respectively, and \(F^{-1}\) denotes the inverse cumulative probability density function.
Figure 2: Time-varying Rank Correlation for High versus Low Portfolios

Panel A: US Stock Market

Beta Portfolio

Coskewness Portfolio

Cokurtosis Portfolio

Panel B: UK Stock Market

Beta Portfolio

Coskewness Portfolio

Cokurtosis Portfolio

Panel C: FX Market

Order-flow Portfolio

Note: This figure shows the rolling window rank correlation (black solid line) with pointwise bootstrapped 90% confidence interval (two red solid lines), including ($BETA_1$,$BETA_5$), ($COSK_1$,$COSK_5$), ($COKT_1$,$COKT_5$), ($OF_1$,$OF_5$). We use 250-day (one trading year) rolling sample for equity portfolio pairs and 60-week rolling sample for currency portfolios pair.
Note: This figure shows the threshold correlation (or exceedance correlation) between high beta (coskewness, cokurtosis and order flow) portfolio and low beta (coskewness, cokurtosis and order flow) portfolio. The threshold correlation measures the linear correlation between two assets when both assets increase or decrease more than specified quantiles (see Longin and Solnik, 2001; Ang and Bekaert, 2002; Ang and Chen, 2002). A solid blue line denotes (9) and a solid red line denotes (10), respectively.
Figure 4: Quantile Dependence between the Standardized Residuals of High and Low Portfolios

Panel A: US Stock Market

Panel B: UK Stock Market

Panel C: FX Market

Note: This figure presents the estimated quantile dependence between the standardized residuals for high beta (coskewness, cokurtosis and order flow) portfolio and low beta (coskewness, cokurtosis and order flow), and the difference in upper and lower quantile dependence. The red dash lines are 90% bootstrap confidence interval for the dependence and lower-upper difference. A solid black line denotes a quantile dependence and dot red lines denote 90% bootstrap confidence interval.
Figure 5: Time-varying Asymmetric Tail Dependence

Panel A: US Stock Market

Beta Portfolio

Coskewness Portfolio

Cokurtosis Portfolio

Panel B: UK Stock Market

Beta Portfolio

Coskewness Portfolio

Cokurtosis Portfolio

Panel C: FX Market

Order-flow Portfolio

Note: This figure shows the dynamic evolution of tail dependence coefficient (TDC) estimated by rotated Gumbel copula from rolling window with window length of 1,000 observations for equity portfolios and 250 observations for FX portfolios. The TDCs between equity portfolios generally increase over time, especially during recent financial crisis. In the US market, lower tail dependence (LTD) is relatively close to or even lower than the upper tail dependence before the financial crisis. However, the LTD has become greater than upper tail dependence (UTD) since the outbreak of the US subprime mortgage crisis in 2007. In the UK market, the LTD is always greater than UTD. For FX portfolios, things are quite different. The LTD substantially decreased from 2006 to 2010 and increased from 2011 to 2012. And the UTD is relatively stable except during the financial crisis. Note that DIFF denotes the difference between LTD and UTD (DIFF = UTD − LTD).
Table 1: Definitions of Portfolios
This table describes the 14 HML portfolios that we constructed for the purpose of empirical analysis in our study. Portfolios are sorted by ex ante market beta, coskewness, cokurtosis and customer order flow. Equity portfolios are annually rebalanced and FX portfolios are weekly rebalanced.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Market</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>HML(Beta,L/S;US)</td>
<td>US Stock Market</td>
<td>Long (short) BETA5 and short (long) BETA1</td>
</tr>
<tr>
<td>HML(Cosk,L/S;US)</td>
<td></td>
<td>Long (short) COSK5 and short (long) COSK1</td>
</tr>
<tr>
<td>HML(Cokt,L/S;US)</td>
<td></td>
<td>Long (short) COKT5 and short (long) COKT1</td>
</tr>
<tr>
<td>HML(Beta,L/S;UK)</td>
<td>UK Stock Market</td>
<td>Long (short) BETA5 and short (long) BETA1</td>
</tr>
<tr>
<td>HML(Cosk,L/S;UK)</td>
<td></td>
<td>Long (short) COSK5 and short (long) COSK1</td>
</tr>
<tr>
<td>HML(Cokt,L/S;UK)</td>
<td></td>
<td>Long (short) COKT5 and short (long) COKT1</td>
</tr>
<tr>
<td>HML(OF,L/S)</td>
<td>FX Market</td>
<td>Long (short) OF5 and short (long) OF1</td>
</tr>
</tbody>
</table>
Table 2: Descriptive Statistics for Returns on the Characteristic-sorted Portfolios

Panel A reports descriptive statistics for daily returns on the characteristic-sorted portfolios from January 4, 2001 to December 31, 2012, which correspond to a sample of 3,268 observations for US market and a sample of 3,283 observations for UK market. We sort stocks into quintiles according to ex ante market beta (coskewness, cokurtosis) and form five capitalization-weighted, annually rebalanced portfolios. The third part of Panel A shows descriptive statistics for weekly returns on the foreign exchange portfolios sorted on their total order flow. We construct five portfolios for total order flows of ten major currency pairs (EUR/USD, JPY/USD, GBP/USD, CHF/USD, AUD/USD, NZD/USD, CAD/USD, SGD/USD, KRW/USD and HKD/USD). Portfolios are rebalanced at the end of every week and they are ranked from low to high order flow; portfolio 1 contains the currencies with the lowest order flow, and portfolio 5 contains the currencies with the highest order flow. Panel B reports descriptive statistics for daily returns on the high minus low (HML) portfolios. LB test lag 5 and LB test lag 10 denote the p-values of the Ljung-Box Q-test for autocorrelation at lags 5 and 10 respectively. We use * or ** to indicate significance at the 5% and 1% levels, respectively.

### Panel A: Characteristic-sorted Portfolio

<table>
<thead>
<tr>
<th>Statistics</th>
<th>US Stock Market</th>
<th>UK Stock Market</th>
<th>FX Market</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BETA1</td>
<td>BETA5</td>
<td>CO5K</td>
</tr>
<tr>
<td>Mean</td>
<td>0.023</td>
<td>-0.021</td>
<td>0.004</td>
</tr>
<tr>
<td>Std dev</td>
<td>0.889</td>
<td>2.567</td>
<td>1.525</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.045</td>
<td>-0.067</td>
<td>0.733</td>
</tr>
<tr>
<td>LB test lag 5</td>
<td>0.00**</td>
<td>0.00**</td>
<td>0.00**</td>
</tr>
<tr>
<td>LB test lag 10</td>
<td>0.00**</td>
<td>0.00**</td>
<td>0.00**</td>
</tr>
<tr>
<td>Engle’s test</td>
<td>0.00**</td>
<td>0.00**</td>
<td>0.00**</td>
</tr>
</tbody>
</table>

### Panel B: HML Portfolio

<table>
<thead>
<tr>
<th>Statistics</th>
<th>HML (Beta, L; US)</th>
<th>HML (Cosk, L; US)</th>
<th>HML (Cokt, L; US)</th>
<th>HML (Beta, L; UK)</th>
<th>HML (Cosk, L; UK)</th>
<th>HML (Cokt, L; UK)</th>
<th>HML (OF, L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.044</td>
<td>-0.001</td>
<td>0.035</td>
<td>-0.062</td>
<td>-0.020</td>
<td>-0.003</td>
<td>0.517</td>
</tr>
<tr>
<td>Std dev</td>
<td>2.206</td>
<td>1.312</td>
<td>1.235</td>
<td>2.108</td>
<td>1.152</td>
<td>1.193</td>
<td>1.170</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.369</td>
<td>1.177</td>
<td>0.346</td>
<td>0.089</td>
<td>0.355</td>
<td>-0.041</td>
<td>1.995</td>
</tr>
<tr>
<td>LB test lag 5</td>
<td>0.00**</td>
<td>0.00**</td>
<td>0.00**</td>
<td>0.01</td>
<td>0.00**</td>
<td>0.00**</td>
<td>0.00**</td>
</tr>
<tr>
<td>LB test lag 10</td>
<td>0.00**</td>
<td>0.01*</td>
<td>0.00**</td>
<td>0.00**</td>
<td>0.00**</td>
<td>0.00**</td>
<td>0.00**</td>
</tr>
<tr>
<td>Engle’s test</td>
<td>0.00**</td>
<td>0.00**</td>
<td>0.00**</td>
<td>0.00**</td>
<td>0.00**</td>
<td>0.00**</td>
<td>0.00**</td>
</tr>
</tbody>
</table>
Table 3: Parameter Estimates and Goodness of Fit Test for the Univariate Modeling

This table reports parameter estimates and p-values from AR and GJR-GARCH models for conditional mean and conditional variance of portfolio returns. Above estimations are estimated by full sample and used for the asymmetric dependence and dynamic dependence sections. We use * or ** to indicate significance at the 5% and 1% levels, respectively. To do out-of-sample VaR forecasts in Section 5, we perform the out-of-sample forecasting using rolling window and therefore all the parameters are time-varying.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BETA1  BETA5  COSK1  COSK5  COKT1  COKT5</td>
<td>BETA1  BETA5  COSK1  COSK5  COKT1  COKT5</td>
<td>OF1  OF5</td>
</tr>
<tr>
<td>φ₀</td>
<td>0.025  -0.021  0.004  0.006  -0.011  0.029</td>
<td>0.039  -0.023  0.018  -0.001  0.009  0.008</td>
<td>-0.179  0.321</td>
</tr>
<tr>
<td>(0.10)</td>
<td>(0.65)  (0.88)  (0.85)  (0.69)  (0.24)</td>
<td>(0.01*)  (0.57)  (0.47)  (0.96)  (0.68)  (0.75)</td>
<td>(0.06)  (0.00**)</td>
</tr>
<tr>
<td>φ₁</td>
<td>-0.064  -      -0.091  -      -0.085  -</td>
<td>-      -      -      -      -0.045  -</td>
<td>-      -0.060  0.094  -</td>
</tr>
<tr>
<td>(0.00**)</td>
<td>-      -      (0.00**)  -      (0.00**)  -</td>
<td>-      -      -      -      (0.01*)  -</td>
<td>-      (0.00**)  (0.04*)  -</td>
</tr>
<tr>
<td>φ₂</td>
<td>-      -      -      -0.085  -      -0.061  -</td>
<td>-      -      -      -      -      -</td>
<td>-      -      -      -      -      -</td>
</tr>
<tr>
<td>(0.00**)</td>
<td>-      -      -      (0.00**)  -      (0.00**)  -</td>
<td>-      -      -      -      -      -</td>
<td>-      -      -      -      -      -</td>
</tr>
<tr>
<td>ω</td>
<td>0.012  0.048  0.027  0.015  0.021  0.016</td>
<td>0.014  0.036  0.021  0.019  0.014  0.028</td>
<td>0.076  0.091  -      -      -      -</td>
</tr>
<tr>
<td>(0.00**)</td>
<td>(0.00**)  (0.00**)  (0.00**)  (0.00**)  (0.00**)  (0.00**)</td>
<td>(0.00**)  (0.00**)  (0.00**)  (0.00**)  (0.00**)  (0.00**)</td>
<td>(0.04*)  (0.05*)  -      -      -      -</td>
</tr>
<tr>
<td>α</td>
<td>0.018  0.001  0.0235  0.000  0.023  0.008</td>
<td>0.012  0.004  0.031  0.006  0.016  0.047</td>
<td>0.052  0.074  -      -      -      -</td>
</tr>
<tr>
<td>(0.19)</td>
<td>(0.00**)  (0.01**)  (0.49)  (0.05*)  (0.11)</td>
<td>(0.07)  (0.63)  (0.00**)  (0.28)  (0.07)  (0.00**)</td>
<td>(0.02*)  (0.02*)  -      -      -      -</td>
</tr>
<tr>
<td>δ</td>
<td>0.122  0.133  0.104  0.163  0.126  0.179</td>
<td>0.111  0.130  0.092  0.135  0.103  0.142</td>
<td>-      -      -      -      -      -</td>
</tr>
<tr>
<td>(0.00**)</td>
<td>(0.00**)  (0.00**)  (0.00**)  (0.00**)  (0.00**)  (0.00**)</td>
<td>(0.00**)  (0.00**)  (0.00**)  (0.00**)  (0.00**)  (0.00**)</td>
<td>(0.02*)  (0.02*)  -      -      -      -</td>
</tr>
<tr>
<td>β</td>
<td>0.901  0.923  0.903  0.9162  0.903  0.896</td>
<td>0.913  0.924  0.911  0.919  0.925  0.867</td>
<td>0.895  0.896  -      -      -      -</td>
</tr>
<tr>
<td>(0.00**)</td>
<td>(0.00**)  (0.00**)  (0.00**)  (0.00**)  (0.00**)  (0.00**)</td>
<td>(0.00**)  (0.00**)  (0.00**)  (0.00**)  (0.00**)  (0.00**)</td>
<td>(0.04*)  (0.00**)  -      -      -      -</td>
</tr>
<tr>
<td>λ</td>
<td>-0.111  -0.072  -0.246  0.006  -0.099  -0.099</td>
<td>-0.076  -0.054  -0.168  0.025  -0.062  -0.076</td>
<td>-0.193  -0.049  -      -      -      -</td>
</tr>
<tr>
<td>KS</td>
<td>0.61  0.17  0.43  0.11  0.14  0.97</td>
<td>0.77  0.96  0.53  0.53  0.35  0.36</td>
<td>0.93  0.93  -      -      -      -</td>
</tr>
<tr>
<td>CvM</td>
<td>0.33  0.10  0.42  0.10  0.16  1.00</td>
<td>0.87  0.92  0.30  0.40  0.45  0.27</td>
<td>0.77  0.91  -      -      -      -</td>
</tr>
</tbody>
</table>
Table 4: Tests for Time-varying Dependence between High and Low Portfolios
We report the $p$-value from tests for time-varying rank correlation between the high portfolio (e.g. BETA5) and the low portfolio (e.g. BETA1). We use * or ** to indicate significance at the 5% and 1% levels, respectively. Having no a priori dates to consider for the timing of a break, we consider naive tests for breaks at three chosen points in sample period, at $t*/T \in \{0.15, 0.50, 0.85\}$, which corresponds to the dates 10-Dec-2001, 03-Jul-2006, 17-Jan-2011. The ‘Anywhere’ column reports the results of test for dependence break of unknown timing proposed by Andrews (1993). To detect whether the dependence structures between characteristic-sorted portfolios significantly changed after the US and EU crisis broke out, we use 15-Sep-2008 (the collapse of Lehman Brothers) and 01-Jan-2010 (EU sovereign debt crisis) as two break points and the ‘Crisis’ panel reports the results for this test. The ‘AR’ panel presents the results from the ARCH LM test for time-varying volatility proposed by Engle (1982). Under the null hypothesis of a constant conditional copula, we test autocorrelation in a measure of dependence (see Patton, 2012).

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Panel A: Break</th>
<th>Panel B: Crisis</th>
<th>Panel C: AR(p)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.15</td>
<td>0.5</td>
<td>0.85</td>
</tr>
<tr>
<td>US BETA1&amp;5</td>
<td>0.00**</td>
<td>0.00**</td>
<td>0.04*</td>
</tr>
<tr>
<td>US COSK1&amp;5</td>
<td>0.00**</td>
<td>0.03*</td>
<td>0.82</td>
</tr>
<tr>
<td>US COKT1&amp;5</td>
<td>0.02*</td>
<td>0.30</td>
<td>0.67</td>
</tr>
<tr>
<td>UK BETA1&amp;5</td>
<td>0.00**</td>
<td>0.00**</td>
<td>0.17</td>
</tr>
<tr>
<td>UK COSK1&amp;5</td>
<td>0.59</td>
<td>0.03*</td>
<td>0.62</td>
</tr>
<tr>
<td>UK COKT1&amp;5</td>
<td>0.98</td>
<td>0.24</td>
<td>0.36</td>
</tr>
<tr>
<td>FX OF1&amp;5</td>
<td>0.59</td>
<td>0.68</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Table 5: Testing the Significance of the Differences of Exceedence Correlations
This table presents the statistics and $p$-values from a model-free symmetry test proposed in Hong et al. (2007) to examine whether the exceedance correlations between low portfolio (i.e. BETA1) and high portfolio (i.e. BETA5) are asymmetric at all. $p$-values less than 0.1 are in bold. The results show that the exceedance correlations of over half of pairs are statistically asymmetric. The $J$ statistics for testing the null hypothesis of symmetric correlation that $\rho^+ (c) = \rho^- (c)$ can be defined as

$$J_\rho = T (\hat{\rho}^+ - \hat{\rho}^-)' \hat{\Omega} \big( \hat{\rho}^+ - \hat{\rho}^- \big)$$

where $\hat{\Omega} = \sum_{l=1}^{T-1} k (l/p) \hat{\gamma}_l$ and $k$ is a kernel function that assigns a suitable weight to each lag of order $l$, and $p$ is the smoothing parameter or lag truncation order (see Hong et al. (2007) for more details).

<table>
<thead>
<tr>
<th>Portfolios</th>
<th>Panel A: US market</th>
<th>Panel B: UK market</th>
<th>Panel C: FX</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J$ statistic</td>
<td>48.471</td>
<td>40.246</td>
<td>44.363</td>
</tr>
<tr>
<td>$p$-value</td>
<td><strong>0.0645</strong></td>
<td>0.2492</td>
<td>0.1334</td>
</tr>
</tbody>
</table>
Table 6: Estimating Tail Dependence Using Parametric Rotated Gumbel and Student’s t Copula.
This table reports the coefficients of lower tail dependence (LTD) and upper tail dependence (UTD) and the difference between them for all the portfolios pairs. The estimations are calculated by the parametric approach in McNeil et al. (2005). $\lambda^G_L$ and $\lambda^G_U$ denote the lower and upper tail dependence coefficients estimated by rotated Gumbel copula and $\lambda^T_L$ and $\lambda^T_U$ denote the lower and upper tail dependence coefficients estimated by t copula. The $p$-values from the tests that the low tail and upper tail dependence coefficients are computed with 500 bootstrap replications. We use * or ** to indicate significance at the 5% and 1% levels, respectively. Equity portfolios support asymmetric dependence while the FX portfolio strongly supports symmetric dependence.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>LTD</th>
<th>UTD</th>
<th>Difference</th>
<th>p-value</th>
<th>Difference</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda^G_L$</td>
<td>$\lambda^T_L$</td>
<td>$\lambda^G_U$</td>
<td>$\lambda^T_U$</td>
<td>$\lambda^G_L - \lambda^G_U$</td>
<td>$\lambda^T_L - \lambda^T_U$</td>
</tr>
<tr>
<td>US BETA1&amp;5</td>
<td>0.256</td>
<td>0.171</td>
<td>0.099</td>
<td>0.018</td>
<td>0.157</td>
<td>0.153</td>
</tr>
<tr>
<td>US COSK1&amp;5</td>
<td>0.315</td>
<td>0.200</td>
<td>0.192</td>
<td>0.153</td>
<td>0.090</td>
<td>0.12</td>
</tr>
<tr>
<td>US COKT1&amp;5</td>
<td>0.306</td>
<td>0.153</td>
<td>0.216</td>
<td>0.103</td>
<td>0.123</td>
<td>0.141</td>
</tr>
<tr>
<td>UK BETA1&amp;5</td>
<td>0.165</td>
<td>0.104</td>
<td>0.024</td>
<td>0.018</td>
<td>0.090</td>
<td>0.092</td>
</tr>
<tr>
<td>UK COSK1&amp;5</td>
<td>0.297</td>
<td>0.203</td>
<td>0.095</td>
<td>0.062</td>
<td>0.202</td>
<td>0.086</td>
</tr>
<tr>
<td>UK COKT1&amp;5</td>
<td>0.209</td>
<td>0.137</td>
<td>0.088</td>
<td>0.052</td>
<td>0.121</td>
<td>0.082</td>
</tr>
<tr>
<td>FX OF1&amp;5</td>
<td>0.231</td>
<td>0.179</td>
<td>0.198</td>
<td>0.102</td>
<td>0.033</td>
<td>0.077</td>
</tr>
</tbody>
</table>
Table 7: Backtesting: Empirical Coverage Probability

This table reports ECP for each HML portfolio and VaR model. Bias summarises the average deviation of 14 portfolios from the nominal coverage probability, 1%, for each VaR model, and RMSE (Root Mean Square Error) summarises the fluctuation of the deviation across 14 portfolios for each VaR model,

\[
Bias = \frac{1}{14} \sum_{p=1}^{14} (ECP_p - 1\%), \quad RMSE = \sqrt{\frac{1}{14} \sum_{p=1}^{14} (ECP_p - 1\%)^2}.
\]

For the UK portfolios, we estimate the VaR models using 250 business days over the period 3 Jan. 2000 - 15 Dec. 2000, and compute the one-day-ahead forecast of the 99 percent VaR for 18 Dec. 2000. We conduct rolling forecasting by moving forward a day at a time and end with the forecast for 31 Dec. 2012. This generates 3,033 out-of-sample daily forecasts. Next we repeat the same process for the US portfolios. It starts with the forecast for 18 Dec. 2000 and ends with the forecast for 31 Dec. 2012. This generates 3,018 out-of-sample daily forecasts. Finally, we repeat the same process for the FX portfolios. It starts with the forecast for 18 Aug. 2006 and ends with the forecast for 23 Mar. 2012. This generates 295 out-of-sample weekly forecasts. D, (P) and (S) denote “Dynamic”, “Parametric” and “Semiparametric”, respectively.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Student’s t Copula</th>
<th>Skew Student’s t Copula</th>
<th>Univariate Models</th>
<th>Multivariate GARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Static D (P) D(S)</td>
<td>Static D (P) D(S)</td>
<td>HS</td>
<td>RM</td>
</tr>
<tr>
<td>HML (Beta, L; US)</td>
<td>0.89% 0.91% 0.86%</td>
<td>0.86% 0.93% 0.86%</td>
<td>1.49% 2.02% 1.66% 1.42% 1.46% 1.39% 1.99%</td>
<td>0.36% 0.46% 0.33%</td>
</tr>
<tr>
<td>HML (Cook, L; US)</td>
<td>1.09% 0.86% 1.06%</td>
<td>1.03% 0.80% 1.03%</td>
<td>1.42% 1.23% 1.19% 1.26% 1.19% 0.63% 1.95%</td>
<td>0.56% 0.80% 0.60%</td>
</tr>
<tr>
<td>HML (Cokt, L; US)</td>
<td>1.03% 0.86% 0.89%</td>
<td>0.99% 0.86% 0.86%</td>
<td>1.19% 1.56% 1.49% 1.59% 1.79% 1.06% 2.02%</td>
<td>1.13% 0.93% 1.03%</td>
</tr>
<tr>
<td>HML (Beta, S; US)</td>
<td>0.56% 1.19% 1.19%</td>
<td>0.56% 1.16% 1.13%</td>
<td>1.26% 1.52% 1.42% 1.39% 1.16% 1.19% 2.29%</td>
<td>1.23% 0.93% 1.06%</td>
</tr>
<tr>
<td>HML (Cook, S; US)</td>
<td>1.26% 0.96% 0.99%</td>
<td>1.19% 0.96% 0.93%</td>
<td>1.33% 2.25% 2.29% 2.05% 1.95% 2.19% 2.95%</td>
<td>2.02% 1.72% 1.69%</td>
</tr>
<tr>
<td>HML (Cokt, S; US)</td>
<td>0.96% 0.86% 0.86%</td>
<td>0.93% 0.83% 0.86%</td>
<td>1.46% 1.76% 1.42% 1.79% 1.82% 1.33% 2.05%</td>
<td>1.23% 1.36% 1.16%</td>
</tr>
<tr>
<td>HML (Beta, L; UK)</td>
<td>1.19% 0.92% 0.86%</td>
<td>1.15% 0.92% 0.92%</td>
<td>1.62% 2.14% 1.65% 1.71% 1.62% 1.65% 2.21%</td>
<td>1.19% 0.63% 1.09%</td>
</tr>
<tr>
<td>HML (Cook, L; UK)</td>
<td>1.19% 1.06% 0.92%</td>
<td>1.09% 1.02% 0.96%</td>
<td>0.92% 0.96% 1.09% 0.99% 1.38% 1.06% 1.91%</td>
<td>0.56% 0.82% 0.46%</td>
</tr>
<tr>
<td>HML (Cokt, L; UK)</td>
<td>0.99% 0.86% 0.99%</td>
<td>0.96% 0.92% 0.99%</td>
<td>1.55% 1.78% 1.71% 1.62% 1.95% 1.48% 2.01%</td>
<td>0.56% 0.66% 0.43%</td>
</tr>
<tr>
<td>HML (Beta, S; UK)</td>
<td>1.02% 1.12% 0.92%</td>
<td>0.99% 1.12% 0.89%</td>
<td>1.55% 1.75% 1.38% 1.45% 1.71% 3.13% 1.62%</td>
<td>1.45% 1.38% 1.75%</td>
</tr>
<tr>
<td>HML (Cook, S; UK)</td>
<td>1.81% 1.19% 1.25%</td>
<td>1.68% 1.12% 1.22%</td>
<td>1.35% 1.95% 1.78% 1.68% 1.91% 2.34% 2.18%</td>
<td>1.68% 1.58% 1.62%</td>
</tr>
<tr>
<td>HML (Cokt, S; UK)</td>
<td>1.22% 0.82% 0.96%</td>
<td>1.12% 0.82% 0.92%</td>
<td>1.22% 1.42% 1.35% 1.35% 1.38% 2.27% 2.01%</td>
<td>1.35% 1.62% 1.38%</td>
</tr>
<tr>
<td>HML (OF, L)</td>
<td>1.02% 1.02% 1.02%</td>
<td>1.02% 0.68% 1.02%</td>
<td>1.71% 1.71% 1.71% 1.71% 2.39% 0.34% 1.37%</td>
<td>1.37% 1.37% 1.37%</td>
</tr>
<tr>
<td>HML (OF, S)</td>
<td>2.73% 1.37% 1.37%</td>
<td>2.39% 1.37% 1.71%</td>
<td>2.73% 2.39% 1.71% 9.90% 10.24% 0.68% 2.39%</td>
<td>2.05% 3.07% 3.07%</td>
</tr>
<tr>
<td>Bias</td>
<td>0.21% 0.00% 0.01%</td>
<td>0.14% -0.04% 0.00%</td>
<td>0.49% 0.74% 0.56% 1.14% 1.28% 0.48% 1.07%</td>
<td>0.20% 0.24% 0.22%</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.56% 0.16% 0.16%</td>
<td>0.45% 0.18% 0.15%</td>
<td>0.64% 0.84% 0.63% 2.52% 2.65% 0.91% 1.13%</td>
<td>0.57% 0.71% 0.74%</td>
</tr>
</tbody>
</table>
This table reports BPZ for each HML portfolio and counts the number of portfolios for each zone. BPZ counts the number of failures of the 99 percent VaR in the previous 250 VaR forecasts. Up to four failures, on average, the portfolio falls into the range of a “Green” zone. If the failures are five or more, the portfolio falls into a “Yellow” (5–9) or “Red” (10+) zone. The VaR model of which BPZ is “Green” zone is preferred. For the UK portfolios, we estimate the VaR models using 250 business days over the period 3 Jan. 2000 - 15 Dec. 2000, and compute the one-day-ahead forecast of the 99 percent VaR for 18 Dec. 2000. We conduct rolling forecasting by moving forward a day at a time and end with the forecast for 31 Dec. 2012. This generates 3,033 out-of-sample daily forecasts. Next we repeat the same process for the US portfolios. It starts with the forecast for 18 Dec. 2000 and ends with the forecast for 31 Dec. 2012. This generates 3,018 out-of-sample daily forecasts. Finally, we repeat the same process for the FX portfolios. It starts with the forecast for 18 Aug. 2006 and ends with the forecast for 23 Mar. 2012. This generates 295 out-of-sample weekly forecasts. D, (P), (S) and G/Y/R denote “Dynamic”, “Parametric”, “Semiparametric” and “Green/Yellow/Red”, respectively.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Student’s t Copula</th>
<th>Skew Student’s t Copula</th>
<th>Univariate Models</th>
<th>Multivariate GARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Static D(P) D(S)</td>
<td>Static D(P) D(S)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HML(Beta, L; US)</td>
<td>Green Green Green</td>
<td>Green Green Green</td>
<td>Yellow Yellow Yellow</td>
<td>Green Green Green</td>
</tr>
<tr>
<td>HML(Cosk, L; US)</td>
<td>Green Green Green</td>
<td>Green Green Green</td>
<td>Green Green Green</td>
<td>Green Green Green</td>
</tr>
<tr>
<td>HML(Beta, S; US)</td>
<td>Green Green Green</td>
<td>Green Green Green</td>
<td>Green Yellow Yellow</td>
<td>Green Green Yellow</td>
</tr>
<tr>
<td>HML(Cosk, S; US)</td>
<td>Green Green Green</td>
<td>Green Green Green</td>
<td>Green Green Yellow</td>
<td>Green Green Yellow</td>
</tr>
<tr>
<td>HML(Cot, S; US)</td>
<td>Green Green Green</td>
<td>Green Green Green</td>
<td>Yellow Yellow Yellow</td>
<td>Yellow Yellow Yellow</td>
</tr>
<tr>
<td>HML(Beta, L; UK)</td>
<td>Green Green Green</td>
<td>Green Green Green</td>
<td>Yellow Yellow Green</td>
<td>Green Green Green</td>
</tr>
<tr>
<td>HML(Cosk, L; UK)</td>
<td>Green Green Green</td>
<td>Green Green Green</td>
<td>Green Green Green</td>
<td>Green Green Green</td>
</tr>
<tr>
<td>HML(Beta, S; UK)</td>
<td>Green Green Green</td>
<td>Green Green Green</td>
<td>Yellow Yellow Green</td>
<td>Green Green Yellow</td>
</tr>
<tr>
<td>HML(Cosk, S; UK)</td>
<td>Yellow Green Green</td>
<td>Yellow Green Green</td>
<td>Green Yellow Yellow</td>
<td>Yellow Yellow Yellow</td>
</tr>
<tr>
<td>HML(Cot, S; UK)</td>
<td>Green Green Green</td>
<td>Green Green Green</td>
<td>Green Green Green</td>
<td>Green Green Green</td>
</tr>
<tr>
<td>HML(OF, L)</td>
<td>Green Green Green</td>
<td>Green Green Green</td>
<td>Yellow Yellow Yellow</td>
<td>Yellow Yellow Yellow</td>
</tr>
<tr>
<td>HML(OF, S)</td>
<td>Yellow Green Green</td>
<td>Yellow Green Yellow</td>
<td>Yellow Yellow Yellow</td>
<td>Yellow Yellow Yellow</td>
</tr>
</tbody>
</table>

| G/Y/R | 12/2/0 | 14/0/0 | 14/0/0 | 12/2/0 | 14/0/0 | 14/0/0 | 8/6/0 | 4/10/0 | 8/6/1 | 7/5/1 | 6/7/1 | 10/4/0 | 1/13/0 | 11/3/0 | 10/4/0 | 10/4/0 |
Table 9: Bactesting: Conditional Coverage Test

This table presents the CC results. The CC test uses the LR statistic and it follows the Chi-squared distribution with two degrees-of-freedom under the null hypothesis. For the UK portfolios, we estimate the VaR models using 250 business days over the period 3 Jan. 2000 - 15 Dec. 2000, and compute the one-day-ahead forecast of the 99 percent VaR for 18 Dec. 2000. We conduct rolling forecasting by moving forward a day at a time and end with the forecast for 31 Dec. 2012. This generates 3,033 out-of-sample daily forecasts. Next we repeat the same process for the US portfolios. It starts with the forecast for 18 Dec. 2000 and ends with the forecast for 31 Dec. 2012. This generates 3,018 out-of-sample daily forecasts. Finally, we repeat the same process for the FX portfolios. It starts with the forecast for 18 Aug. 2006 and ends with the forecast for 23 Mar. 2012. This generates 295 out-of-sample weekly forecasts. * indicates that the VaR model is rejected at the 5% significance level. D, (P) and (S) denote “Dynamic”, “Parametric” and “Semiparametric”, respectively.

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<tr>
<th>Portfolio</th>
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<th>Skew Student’s t Copula</th>
<th>Univariate Models</th>
<th>Multivariate GARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Static D (P) D (S)</td>
<td>Static D (P) D (S)</td>
<td>HS RM GARCH GJR EGARCH FEVT CAViaR</td>
<td>BEKK CCC DCC</td>
</tr>
<tr>
<td>HML(Beta, L; US)</td>
<td>0.84 0.69 1.06</td>
<td>1.06 2.79 3.44</td>
<td>14.54* 26.46* 12.67* 6.11* 5.79</td>
<td>5.36 27.48*</td>
</tr>
<tr>
<td>HML(Cokt, L; US)</td>
<td>13.07* 6.07* 8.33*</td>
<td>13.83* 7.43* 8.60*</td>
<td>9.90* 4.43 1.94 2.87</td>
<td>1.64 5.05 29.26*</td>
</tr>
<tr>
<td>HML(Cokt, L; US)</td>
<td>8.60* 11.25* 5.52</td>
<td>8.96* 6.07* 6.07*</td>
<td>12.53* 12.26* 8.19*</td>
<td>10.47* 21.18* 21.8* 0.80 24.97*</td>
</tr>
<tr>
<td>HML(Beta, S; US)</td>
<td>7.09* 1.94 4.22</td>
<td>8.28* 1.56 4.01</td>
<td>8.22* 11.59* 6.92*</td>
<td>4.43 1.38 14.62 40.05*</td>
</tr>
<tr>
<td>HML(Cokt, S; US)</td>
<td>8.22* 1.20 1.05</td>
<td>7.97* 1.20 1.42</td>
<td>5.42 38.27* 37.26*</td>
<td>26.37* 24.11* 32.26* 83.84*</td>
</tr>
<tr>
<td>HML(Cokt, S; US)</td>
<td>0.61 1.06 1.06</td>
<td>0.69 1.37 1.06</td>
<td>7.53* 15.21* 6.11*</td>
<td>16.28* 17.41* 29.26* 0.78 7.22*</td>
</tr>
<tr>
<td>HML(Beta, L; UK)</td>
<td>1.88 0.87 1.10</td>
<td>1.51 0.71 0.71</td>
<td>9.85* 30.31* 10.83*</td>
<td>12.91* 9.85* 10.83* 36.45* 27.48*</td>
</tr>
<tr>
<td>HML(Beta, S; UK)</td>
<td>4.18 0.78 0.71</td>
<td>3.99 0.66 0.62</td>
<td>0.71 0.62 1.02</td>
<td>1.06 6.26* 14.62 29.26* 7.22* 26.37 11.22*</td>
</tr>
<tr>
<td>HML(Cokt, L; UK)</td>
<td>1.06 1.10 0.60</td>
<td>1.22 0.71 0.74</td>
<td>15.50* 24.21* 14.71*</td>
<td>11.41* 23.65* 23.65* 7.60* 26.17* 36.45* 27.48*</td>
</tr>
<tr>
<td>HML(Cokt, S; UK)</td>
<td>0.66 1.21 0.71</td>
<td>0.60 1.21 0.87</td>
<td>9.50* 17.08* 4.31</td>
<td>4.31 10.30* 16.14* 93.32* 16.78* 24.97*</td>
</tr>
<tr>
<td>HML(Cokt, S; UK)</td>
<td>16.35* 1.88 2.78</td>
<td>11.85* 1.51 2.30</td>
<td>8.97* 22.01* 16.06*</td>
<td>11.85* 20.15* 40.14* 37.49* 11.85* 10.30* 11.15* 26.17*</td>
</tr>
<tr>
<td>HML(Cokt, S; UK)</td>
<td>4.38 1.42 0.62</td>
<td>3.99 1.42 0.71</td>
<td>1.90 4.96 4.55</td>
<td>3.72 5.24 36.73* 24.21* 4.55 16.14* 6.26*</td>
</tr>
<tr>
<td>HML(OF, L)</td>
<td>0.06 0.06 0.06</td>
<td>0.06 0.36 0.06</td>
<td>1.41 1.41 1.41</td>
<td>1.41 4.48 1.72 0.47 0.47 0.47 0.47</td>
</tr>
<tr>
<td>HML(OF, S)</td>
<td>6.51* 0.47</td>
<td>0.47</td>
<td>4.48 0.47 0.47</td>
<td>4.48 85.34* 90.48* 0.36 4.48 27.48* 8.80* 9.47*</td>
</tr>
</tbody>
</table>

# of Rej | 6 2 1 | 5 2 2 | 10 9 8 | 9 9 6 | 12 7 8 | 10
Table 10: Backtesting: Dynamic Quantile Test

This table presents the DQ test results. The DQ test uses the Wald statistic and it follows the Chi-squared distribution with 6 degrees-of-freedom under the null hypothesis (see Eq. (26)). For the UK portfolios, we estimate the VaR models using 250 business days over the period 3 Jan. 2000 - 15 Dec. 2000, and compute the one-day-ahead forecast of the 99 percent VaR for 18 Dec. 2000. We conduct rolling forecasting by moving forward a day at a time and end with the forecast for 31 Dec. 2012. This generates 3,033 out-of-sample daily forecasts. Next we repeat the same process for the US portfolios. It starts with the forecast for 18 Dec. 2000 and ends with the forecast for 31 Dec. 2012. This generates 3,018 out-of-sample daily forecasts. Finally, we repeat the same process for the FX portfolios. It starts with the forecast for 18 Aug. 2006 and ends with the forecast for 23 Mar. 2012. This generates 295 out-of-sample weekly forecasts. * indicates that the VaR model is rejected at the 5% significance level. D, (P) and (S) denote “Dynamic”, “Parametric” and “Semiparametric”, respectively.

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<thead>
<tr>
<th>Portfolio</th>
<th>Student's $t$ Copula</th>
<th>Skew Student's $t$ Copula</th>
<th>Univariate Models</th>
<th>Multivariate GARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Static</td>
<td>D (P)</td>
<td>D (S)</td>
<td>Static</td>
</tr>
<tr>
<td>HML (Cosk, L; US)</td>
<td>70.84*</td>
<td>16.83*</td>
<td>37.22*</td>
<td>63.83*</td>
</tr>
<tr>
<td>HML (Cokt, L; US)</td>
<td>54.05*</td>
<td>37.76*</td>
<td>16.37*</td>
<td>56.29*</td>
</tr>
<tr>
<td>HML (Cosk, S; US)</td>
<td>24.23*</td>
<td>7.46</td>
<td>5.80</td>
<td>25.28*</td>
</tr>
<tr>
<td>HML (Cokt, S; US)</td>
<td>4.67</td>
<td>7.33</td>
<td>6.76</td>
<td>4.49</td>
</tr>
<tr>
<td>HML (Beta, L; UK)</td>
<td>12.19</td>
<td>6.32</td>
<td>2.87</td>
<td>11.64</td>
</tr>
<tr>
<td>HML (Cosk, L; UK)</td>
<td>42.12*</td>
<td>12.82*</td>
<td>20.17*</td>
<td>42.91*</td>
</tr>
</tbody>
</table>
| HML (Cokt, L; UK)  | 21.36*| 3.32  | 4.67  | 21.15*| 3.98  | 4.68  | 48.47*| 161.30*| 19.81*| 13.39*| 41.77*| 12.76*| 81.08*| 10.95 | 5.07  | 10.17*
| HML (Beta, S; UK)  | 10.17  | 6.07  | 10.25 | 9.76   | 6.10  | 10.05 | 44.47*| 43.65*| 15.18*| 33.28*| 36.41*| 197.49*| 17.88*| 12.25*| 31.35*|
| HML (Cosk, S; UK)  | 50.81*| 8.15  | 13.30*| 39.89*| 6.91  | 10.98 | 28.16*| 56.95*| 31.22*| 19.35*| 36.10*| 79.15*| 170.52*| 25.06*| 24.43*| 29.10*|
| HML (Cokt, S; UK)  | 24.09*| 7.84  | 5.88  | 21.69*| 8.25  | 5.14  | 16.18*| 44.80*| 11.92*| 28.69*| 64.17*| 121.84*| 116.67*| 28.67*| 40.79*| 21.03*|
| HML (OF, L)        | 0.68   | 0.32  | 0.41  | 0.64   | 0.38  | 0.40  | 16.36*| 8.88  | 3.94  | 6.15  | 19.98*| 1.25  | 29.18*| 2.84  | 1.11  | 1.60  |
| HML (OF, S)        | 25.44*| 1.64  | 0.84  | 10.50  | 1.57  | 1.29  | 57.20*| 129.96*| 24.05*| 284.97*| 293.85*| 0.62  | 22.87*| 25.65*| 30.40*| 32.31*|
| # of Rej.          | 8      | 4     | 6     | 7      | 2     | 4     | 13    | 12    | 11    | 11    | 12    | 7     | 14    | 7    | 8    | 12    |